GCE O'LEVEL MATHS

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Yayasan MENDAKI In Quest of Excellence

MATHEMATICS GCE O' Level
EXAMINATION FORMAT

Paper	Duration	Description	Marks	Weighting
Paper 1	2 hours	There will be about 25 short answer questions testing more on the fundamental skills and concepts. Candidates are to answer ALL questions.	80	50%
Paper 2	2 ½ hours	There will be about 10 to 11questions of varying marks and lengths testing more on higher order thinking skills. Candidates are to answer ALL questions.	100	50%

NOTES

1. Omission of essential working will result in loss of marks.
2. Some questions may integrate ideas from more than one topic of the syllabus where applicable.
3. Relevant mathematical formulae will be provided for candidates.
4. Scientific calculators are allowed in both Paper 1 and Paper 2.
5. Candidates should also have geometrical instruments with them for Paper 1 and Paper 2.
6. Unless stated otherwise within a question, three-figure accuracy will be required for answers. This means that four-figure accuracy should be shown throughout the working, including cases where answers are used in subsequent parts of the question. Premature approximation will be penalised, where appropriate. Angles in degrees should be given to one decimal place.
7. SI units will be used in questions involving mass and measures. Both the 12-hour and 24-hour clock may be used for quoting times of the day. In the 24-hour clock, for example, 3.15 a.m. will be denoted by 0315; 3.15 p.m. by 1515, noon by 1200 and midnight by 2400.
8. Candidates are expected to be familiar with the solidus notation for the expression of compound units, e.g. 5 cm/s for 5 centimetres per second, 13.6 g/cm3 for 13.6 grams per cubic centimetre.
9. Unless the question requires the answer in terms of π, the calculator value for π or π = 3.142 should be used.
10. Spaces will be provided on the question paper of Paper 1 for working and answers.

Mathematical Formulae

Compound Interest

Total amount = $P\left(1+\frac{r}{100}\right)^n$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$ where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

Statistics

Mean = $\frac{\sum fx}{\sum f}$

Standard deviation = $\sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$

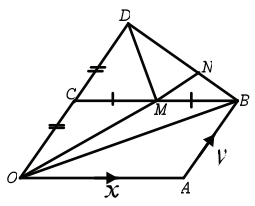
Sample 'O' Level Maths Questions And Tips

Vectors

(A) The coordinates of point A is (3, 4) and point B is (12, 10).
If point P lies on the line AB such that $3AP = AB$.

(i) Express $\overrightarrow{AB}$ as a column vector.
(ii) Find $|\overrightarrow{AB}|$.
(iii) Find the coordinates of P.

(B)



OABC is a parallelogram and M is the midpoint of BC.
OC is produced to D so that OC = CD. OM is produced to meet BD at N.
Given that $\overrightarrow{OA}$= x and $\overrightarrow{AB}$= y, express the following vectors in terms of x and y.

(i) $\overrightarrow{OB}$, (iii)$\overrightarrow{BD}$
(ii) $\overrightarrow{OM}$
(iv) Given that $ON=\frac{4}{3}OM$, find the numerical value of
(a) $\frac{\text{area of }\Delta MDN}{\text{area of }\Delta MDO}$,
(b) $\frac{\text{area of trapezium } OABD}{\text{area of }\Delta OAB}$

Qn	Worked Solution	Tips	Common Mistakes
A(i)	$\overrightarrow{AB}=\overrightarrow{AO}+\overrightarrow{OB}$ $=\begin{pmatrix}-3\\-4\end{pmatrix}+\begin{pmatrix}12\\10\end{pmatrix}$ $=\begin{pmatrix}9\\6\end{pmatrix}$	- It is good to begin with *A* and ends with *B*. Meaning, start from point *A* to *O*, followed by *O* to *B*. In this manner you will always remember where to start and end.	$\overrightarrow{AO}=\begin{pmatrix}-4\\-3\end{pmatrix}$. Here, students not only change the signs but also swap the position of the numbers.
A(ii)	$\lvert\overrightarrow{AB}\rvert=\sqrt{9^2+6^2}$ = 10.8 (3sf)	Do ensure the answer written is in 3 sf since the actual value is not exact.	$\sqrt{9^2-6^2}$. Students are confused with the formula of finding the length of the two coordinates.
A(iii)	$\overrightarrow{OP}=\overrightarrow{OA}+\overrightarrow{AP}$ $=\overrightarrow{OA}+\frac{1}{3}\overrightarrow{AB}$ $=\begin{pmatrix}3\\4\end{pmatrix}+\frac{1}{3}\begin{pmatrix}9\\6\end{pmatrix}$ $=\begin{pmatrix}6\\6\end{pmatrix}$ Coordinates of *P* is (6, 6)	Same as A(i).	Some students will leave the answer as $\begin{pmatrix}6\\6\end{pmatrix}$. They will lose marks as the question is asking for coordinates not vector form. In other words, the answer should be in (6, 6) form.
B(i)	$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}$ $=\boldsymbol{x}+\boldsymbol{y}$	Same as A(i).	
B(ii)	$\overrightarrow{OM}=\overrightarrow{OB}+\overrightarrow{BM}$ $=\overrightarrow{OB}+\frac{1}{2}\overrightarrow{AO}$ $=\boldsymbol{x}+\boldsymbol{y}+\frac{1}{2}(-\boldsymbol{x})$ $=\frac{1}{2}\boldsymbol{x}+\boldsymbol{y}$	Take note of the direction of $\overrightarrow{BM}$. It is in the opposite direction of $\overrightarrow{OA}$. In other words, you are required to change the signs	Some students do not change the sign for $\overrightarrow{AO}$ in their working.
B(iii)	$\overrightarrow{BD}=\overrightarrow{BC}+\overrightarrow{CD}$ $=-\boldsymbol{x}+\boldsymbol{y}$	Take note that $\overrightarrow{BC}$ is parallel to $\overrightarrow{AO}$ since it is a parallelogram. And $\overrightarrow{CD}$ is in the same direction and same magnitude as $\overrightarrow{OC}$.	Same as B(ii). Students do not change the sign for $\overrightarrow{BC}$.

B(iv)a	$\frac{\text{area of }\Delta MDN}{\text{area of }\Delta MDO}=\frac{\frac{1}{2}\times MN\times\perp ht}{\frac{1}{2}\times OM\times\perp ht}$ $=\frac{1}{3}$	Compare the two triangles and identify the common perpendicular height. Then, put as ratio, the length of their bases.	Students tend to compare the height instead, or they ratio the wrong sides of the triangle.
B(iv)b	$\frac{\text{area of trapezium } OABD}{\text{area of }\Delta OAB}$ $=\frac{\frac{1}{2}\times(OD+AB)\times\perp ht}{\frac{1}{2}\times AB\times\perp ht}$ $=\frac{3}{1}$ = 3	Observe that OABD is a combination of two triangles, OAB and OBD. Once youcan see, that, identify the common perpendicular height.	Students cannot see the relationship of triangle OAB and the trapezium, often resulting in leaving the question blank.

Algebra

(A) Express as a single fraction in its simplest form

$$\frac{1}{x-2}+\frac{1}{2+5x-3x^2}$$

(B) It is given that $y=b\sqrt{\frac{3-x}{a}}$.

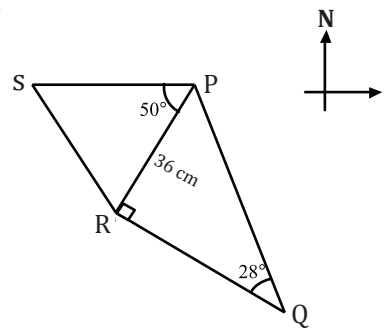
Express x in terms of y, a and b.

(C) Solve $(3x-2)^2=16$.

(D) Simplify $\frac{4-p^2}{3p-6}$.

Qn	Worked Solution	Tips	Common Mistakes
A	$\frac{1}{x-2}+\frac{1}{2+5x-3x^2}$ $=\frac{1}{(x-2)}+\frac{1}{(1+3x)(2-x)}$ $=\frac{-1}{(2-x)}+\frac{1}{(1+3x)(2-x)}$ $=\frac{-1(1+3x)+1}{(1+3x)(2-x)}$ $=\frac{-3x}{(1+3x)(2-x)}$ or $\frac{3x}{(1+3x)(x-2)}$	In these types of questions, it is almost certain that one or both denominator can be factorised. Once factorised, you will notice that there will be a common factor.	In trying to change $(x-2)$ to $-(2-x)$, some students may make mistakes such as missing the negative for x in $(2+x)$ or missing the negative outside the bracket in $(2-x)$.
B	$y=b\sqrt{\frac{3-x}{a}}$ $\frac{y}{b}=\sqrt{\frac{3-x}{a}}$ $\frac{y^2}{b^2}=\frac{3-x}{a}$ $\frac{ay^2}{b^2}=3-x$ $x=3-\frac{ay^2}{b^2}\left(\text{or }\frac{3b^2-ay^2}{b^2}\right)$	*Bring over to the other side of the equation the terms furthest away from the subject. In this case, you will see that *b* is the term that is furthest away from *x*, followed by $\sqrt{}$, *a*, and lastly 3.	Students are not able to follow the order as mentioned in *(Tips).
C	$(3x-2)^2=16$ $3x-2=\pm\sqrt{16}$ $3x-2=4$ or -4 $x=2$ or $-\frac{2}{3}$	There are two ways of solving this question. One way is through expanding $(3x-2)^2$, followed by solving quadratic equations. The other way is by using the method stated on the left.	Students often state the positive square rooted answer only, leaving out the negative value.
D	$\frac{4-p^2}{3p-6}$ $=\frac{(2+p)(2-p)}{3(p-2)}$ $=\frac{-(2+p)(p-2)}{3(p-2)}$ $=\frac{-(2+p)}{3}$ or $\left(\frac{-2-p}{3}\right)$	Always try to see whether the numerator and the denominator can be factorised. Most of the time, they can be factorised.	$4-p^2$ is factorised wrongly to become $(2-p)(2-p)$.

Trigonometry



In the diagram, P, Q, R and S are four markers in a horizontal field.
Given that PS = 40m, PR = 36m, $R\hat{P}S = 50°$, $P\hat{Q}R = 38°$, $P\hat{R}Q = 90°$ and S is due west of P.

(A) Calculate
(i) the bearing of P from R,
(ii) the bearing of P from Q,
(iii) PQ,
(iv) RS,
(v) the shortest distance from S to PR.

(B) A vertical pole of height 7.5m is planted at S. A man walks along the path PR. Find the greatest angle of elevation of the top of the pole when the man viewed from any point along the path.

Qn	Worked Solution	Tips	Common Mistakes
A(i)	Bearing of *P* from *R* = 090° − 050° = 040°	Do take note of the direction of North. Occasionally, North is not pointed upwards but towards some other direction.	Students leave the answer as 40°. But, bearing must always be in 3 digits.
A(ii)	$R\hat{P}Q = 90° - 38°$ = 52° Angle from North to line *PQ* = 360° − 52° − 50° − 90° = 168° Bearing of *P* from *Q* = 360° − (180° − 168°) = 348°	Draw the North lines at *P* and *Q*. You will see that they are parallel and this will help you in solving the question.	Nil
A(iii)	$\frac{36}{PQ}=\sin 38°$ $PQ=\frac{36}{\sin 38°}$ = 58.5m	Use the trigonometric ratios for right - angled triangle.	In cross multiplication, students tend to make this mistake: $PQ = 36\sin 38°$.
A(iv)	$RS^2=40^2+36^2-2(40)(36)\cos 50°$ = 1044.772 *RS* = 32.3m	Use sine or cosine rule in questions involving triangles that are not right-angled triangle. For this question, only 2 lengths and an angle are included. Thus, only cosine rule can be used.	In part solving calculation, students tend to make an error in calculating $40^2+36^2-2(40)(36)$ together, multiplying $\cos 50°$. Students also have the tendency to forget to square root RS^2 to get *RS*.

A(v)	$\frac{\text{Shortest Dist}}{40}=\sin 50°$ Shortest Dist = $40\sin 50°$ = 30.64m = 30.6m (3 sf)	The shortest distance is the perpendicular line to *PR* from *S*.	Wrong usage of cosine and sine ratios.
B	$\tan x=\frac{7.5}{30.6}$ $x=\tan^{-1}\left(\frac{7.5}{30.6}\right)$ = 13.75° = 13.8° (1 dp)	The greatest angle of elevation happens when the horizontal distance is the shortest (from part A(v)	In pressing the calculator, students tend to mistakenly calculate ($\tan^{-1}$ 7.5) followed by the division of 30.6.

Quadratic Equations

The distance between two towns, P and Q, is 180 km. Mr Lim drove from P to Q at an average speed of v km/h. After traveling for 100 km, he increases his speed by 6 km/h and maintains this speed for the rest of the journey.

(A) Write down an expression, in terms of v, for
(i) the number of hours taken to travel the first 100 km,
(ii) the number of hours to travel the rest of the journey.

(B) The first part of the journey took 20 minutes more than the second part of the journey. Write down an equation in v, and show that it reduces to

$$v^2-54v-1800=0$$

(C) Solve the equation $v^2-54v-1800=0$, giving both answers to 2 decimal places.

(D) Calculate, correct to the nearest minutes, the time he took to travel from P to Q.

Qn	Worked Solution	Tips	Common Mistakes
A(i)	$\frac{100}{v}$ hours	The answer is most likely in fractions, where the variable, *v*, will be the denominator.	Nil
A(ii)	$\frac{80}{v+6}$ hours	Same as A(i)	Students do not add the six at the denominator as shown, $\frac{80}{v}+6$.
B	$\frac{100}{v}-\frac{80}{v+6}=\frac{20}{60}$ $\frac{100(v+6)-80v}{v(v+6)}=\frac{1}{3}$ $\frac{20v+600}{v^2+6v}=\frac{1}{3}$ $v^2+6v=60v+1800$ $v^2-54v-1800=0$	Always consider the longer time to be subtracted by the shorter time.	Students tend to forget to change 20 minutes to hours. Another mistake is that students tend to subtract the wrong expression as shown: $\frac{80}{v+6}-\frac{100}{v}=\frac{20}{60}$
C	$v=\frac{54\pm\sqrt{54^2-4(1)(-1800)}}{2(1)}$ $=\frac{54+\sqrt{10116}}{2}$ or $\frac{54-\sqrt{10116}}{2}$ = 77.29 or − 23.29(rejected)	You need to show the expression after substituting the numbers as shown on the left. Do reject an answer that is not applicable to the question.	Students use the calculator to solve the equation. Marks will be deducted if the first step is not shown. Students fail to convert the answer to the decimal place(s) as required in the question.
D	Total time taken $=\frac{100}{77.29}+\frac{80}{77.29+6}$ = 2.254 hours = 135 minutes	Nil	Students fail to convert the answer to minutes as required.

Matrices

A sports shop sells two brands of t-shirts, Niki and Adi.
Each brand is sold in three different sizes, small, medium and large.
The sales in two successive weeks are given in the tables below.

	First Week			Second Week		
	Small	Medium	Large	Small	Medium	Large
Niki	16	28	5	10	20	15
Adi	10	36	12	16	32	8

The information for the first week's sales can be represented by the matrix

$$\mathbf{P}=\begin{pmatrix}16&28&5\\10&36&12\end{pmatrix}$$

The information for the second week is represented by a matrix **Q**.

(A) Write down the matrix Q.

(B) Calculate $\frac{1}{2}(\mathbf{P}+\mathbf{Q})$

(C) Describe what is represented by the elements of $\frac{1}{2}(\mathbf{P}+\mathbf{Q})$

(D) Given that $\mathbf{X}=\begin{pmatrix}1\\1\\1\end{pmatrix}$, calculate $\mathbf{S}=\mathbf{PX}$.

(E) Describe what is represented by the elements of S.

(F) Given that $\mathbf{Y}=\begin{pmatrix}1&1\end{pmatrix}$, calculate $\mathbf{V}=\mathbf{YS}$.

(G) Describe what is represented by the elements of **V**.

Qn	Worked Solution	Tips	Common Mistakes
A	$\mathbf{Q}=\begin{pmatrix}10&20&15\\16&32&8\end{pmatrix}$.	Usually you need to observe how the table is presented, and then write it down as required.	NIL
B	$\frac{1}{2}(\mathbf{P}+\mathbf{Q})$ $=\frac{1}{2}\left(\begin{pmatrix}16&28&5\\10&36&12\end{pmatrix}+\begin{pmatrix}10&20&15\\16&32&8\end{pmatrix}\right)$ $=\frac{1}{2}\begin{pmatrix}26&48&20\\26&68&20\end{pmatrix}$ $=\begin{pmatrix}13&24&10\\13&34&10\end{pmatrix}$	Add (**P** + **Q**) first before including the $\frac{1}{2}$. You need to simplify the matrix where possible.	Students multiply the $\frac{1}{2}$ to the matrix **P** only but not **Q**.
C	Average sales of each type of t-shirts for each brand.	Be clear in your explanation. Use the words "each" effectively.	Students are not able to explain in a manner that is acceptable.
D	**S** = **PX** $=\begin{pmatrix}16&28&5\\10&36&12\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}$ $=\begin{pmatrix}49\\58\end{pmatrix}$	**P** is a **2**x3 matrix and **X** is a 3x**1** matrix. The answer will be a **2x1** matrix.	Students are not sure on how to multiply matrices. Or they do not get a **2x1** matrix.
E	Number of Niki and Adi t-shirts sold in the first week.	Same as (C)	Same as (C)
F	**V** = **YS** $=\begin{pmatrix}1&1\end{pmatrix}\begin{pmatrix}49\\58\end{pmatrix}$ $=\begin{pmatrix}107\end{pmatrix}$	**Y** is a **1**x2 matrix and **S** is a 2x**1** matrix. The answer will be a **1x1** matrix.	Students multiply wrongly, leaving the answer in a 2x2 matrix.
G	Total number of t-shirts sold in the first week.	Same as (C)	Same as (C)

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