Candidate's Name	Class	Register Number

PRELIMINARY EXAMINATION 2013
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

MATHEMATICS
Paper 1 **4016/01**

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen in the spaces provided on the Question Paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown in the space below that question.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's use:
/80

This question paper consists of 16 printed pages (including this page).

[Turn over

Mathematical Formulae

Compound interest

Total amount $= P(1+\frac{r}{100})^n$

Mensuration

Curved surface area of a cone = $\pi r l$

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

Mean $= \frac{\Sigma f x}{\Sigma f}$

Standard deviation = $\sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2}$

For Examiner's Use

For Examiners Use

Answer **all** the questions.

1 Evaluate $0.68^5 \times 9.85 \times 10^5$, giving your answer in standard form correct to 3 significant figures.

Answer ____________________ [1]

---

2 m is inversely proportional to n^2, where m and n are positive numbers.
If m is increased by 1500%, find the percentage change in n.

Answer ________________________ [3]

---

3 Simplify each of the following expressions as a positive index.

(a) $\sqrt[3]{z^{-6}}$,

(b) $\left(pq^2\right)^{-5} \times \dfrac{q^0}{p^3}$.

Answer *(a)* ____________________ [1]

(b) ____________________ [2]

---

4 Kayla had 130 pens, 90 rulers and 55 erasers. She packed them into pencil boxes with an equal number of the 3 items. After packing, she had 4 pens and 1 eraser not packed.

(a) Find the maximum number of pencil boxes that she had packed.

(b) Kayla wanted to pack another pencil box with the same number of items using the remaining pens and erasers. How many pieces of each of the stationery must she buy?

Answer *(a)* ______________ [2]

(b) ______________________________ [2]

5 Eddy wanted to train for his 2.4 km run on a treadmill machine. He chose a programme that allowed him to run at different speeds over various parts of the distance. For example, the machine was set to a constant speed of 4.0 m/s for the first 400 m, hence he was supposed to run that distance in 100 s.

(a) A table showing the different settings is shown below. Complete the table.

Answer (a) [1]

Distance (metres)	400	1400	400	200
Time (seconds)	100	400	100	
Speed (m/s)	4.0		4.0	5.0

(b) Calculate the average speed for the whole run.

Answer *(b)* ____________ m/s [2]

For Examiners Use

6 The Earth is approximately 0.1496 tetrameters (Tm) away from the Sun. Light takes about 3.336 nanoseconds to travel 1 metre.

(a) Express 0.1496 Tm in km, giving your answers in standard form.

(b) Calculate the time taken, in seconds, for light to travel from the Sun to the Earth.

Answer *(a)* ____________________ [1]

(b) ____________________ [1]

---

7 A map has a scale of 1: *n*, where *n* is a whole number. A lake of area 1 km^2 is represented by an area of 4 cm^2 on this map.

(a) Evaluate the value of *n*.

A new map has to be drawn for the same area, such that the area of the lake on the original map is nine times bigger than the area represented on the new map.

(b) Calculate the distance, in km, represented by 16 cm on the new map.

Answer *(a)* n = ________________ [1]

(b) ________________ km [1]

For Examiners Use

8 **(a)** Factorise $a^2 - b^2$.

(b) **Hence, without using a calculator**, evaluate $\sqrt{8.2^2 - 1.8^2}$.

Answer *(a)* ____________________ [1]

(b) ____________________ [1]

---

9 The first 3 terms in a sequence of numbers, 3, 6, 12, …, are given below.

$w_1 = 3 \times 2^0 = 3$

$w_2 = 3 \times 2^1 = 6$

$w_3 = 3 \times 2^2 = 12$

(a) Write down the fourth and fifth terms.

(b) Write down an expression, in terms of n, for the nth term, w_n, of the sequence.

Answer *(a)* _________ , _________ [1]

(b) ____________________ [1]

---

10 Simplify $\dfrac{3}{3x-2} - \dfrac{13}{6x^2 + 5x - 6}$

Answer ____________________ [3]

For Examiner's Use

11 The school needs to buy new sets of tables and chairs to replace the damaged ones in the hall. The school found that at least 20 sets of tables and chairs need to be replaced. Each chair costs \$90 and each table costs \$60. The budget allocated for the replacement of furniture is \$4,000 for this year.

(a) Form two inequalities in x, the number of sets of tables and chairs to be purchased.

(b) Solve the inequalities and represent your solution on a number line.

(c) Hence, find the maximum sets of tables and chairs that the school can buy.

Answer (a) ________ , ____________ [2]

(b) ________________ [1]

(b)

$\longleftrightarrow$ [1]

(c) ______________ [1]

For Examiner's Use

For Examiner's Use

12 Line L_1 is given by the equation $2y + x = 1$. Line L_2 passes through the point $(6\ ,-8)$ and it has the same gradient as line L_1.

(a) Find the equation of line L_2.

(b) Find the distance from the point $(6\ ,-8)$ to the point $(-2\ ,-2)$.

Answer *(a)* ____________________ [2]

(b) ____________________ [1]

13 Four cards numbered 1 to 4 are put in a box. One card is picked at random and the number is noted. Without replacing the first card, a second card is randomly picked and the number is noted. Find the probability that the sum of both numbers is odd.

Answer ____________________ [2]

14 The table below shows the entrance fees of an adult and a child and their average numbers on each day to the IMAX Movie at the Singapore Science Centre (SSC).

	Adult	Child
Fees	\$10.00	\$5.00
Average number (working day)	200	600
Average number (non-working day)	300	900

(a) Matrices *A* (order is 2×2) and *B* (order is 2×1) contain elements such that the elements of their product *AB* will give the amount of entrance fee collected for each of the working day and non-working day. Calculate this product *AB*.

(b) The management of SSC wanted to give a discount everyday during the December school holidays, reducing the adult and child entrance fee to \$8.00 and \$4.50 respectively. Matrix *C* (order is 2×1) represents the new fees. Calculate *AC*.

(c) Evaluate $D = (AB - AC)$ and explain what the elements of *D* represents.

Answer

(a) $AB =$ ______________ [2]

(b) $AC =$ ______________ [2]

(c) $D =$ ______________ [1]

D represents ____ [1]

For Examiner's Use

15 In a class of 18 students, the marks of their Mathematics test, out of a maximum of 100 marks, is as shown below.

65	82	35	83	50	65
71	60	62	73	61	70
62	77	76	91	67	81

(a) Construct a stem-and-leaf diagram for the above data.

(b) Calculate the mean marks scored.

(c) Calculate the median marks scored.

(d) State the modal marks scored.

Answer (a) [2]

(b) ____________________ [1]

(c) ____________________ [1]

(d) ____________________ [1]

For Examiner's Use

16 Draw a triangle ABC to scale where $AB = 10\ cm$, $AC = 9\ cm$ and $BC = 6\ cm$.
By drawing suitable lines, locate a point P such that P is equidistant from line AB, line BC, point A and point B.

Answer [4]

For Examiner's Use

For Examiner's Use

17 The rate of 6-monthly road tax for cars from 1 July 2008 is shown in the table below.

Engine Capacity (EC) in cc	6-monthly Road Tax Formula
EC $\leq$ 600	[\$200 $\times$ 0.782]
600 < EC $\leq$ 1,000	[\$200 + 0.125 $\times$ (EC - 600)] $\times$ 0.782
1,000 < EC $\leq$ 1,600	[\$250 + 0.375 $\times$ (EC - 1,000)] $\times$ 0.782
1,600 < EC $\leq$ 3,000	[\$475 + 0.75 $\times$ (EC - 1,600)] $\times$ 0.782
EC > 3,000	[\$1,525 + 1 $\times$ (EC - 3,000)] $\times$ 0.782

(a) Calculate the 6-monthly road tax on a car of engine capacity 1,500 cc.

(b) If the annual road tax on a certain car is \$1,446.70, calculate the engine capacity of the car.

Answer *(a)* ________________ [2]

(b) ________________ [2]

18 The frequency table shows the number of goals scored in a series of soccer matches.

Number of goals	0	1	2	3	4	5
Number of matches	7	x	5	3	1	4

(a) If the mean number of goals is more than 2, find the largest possible value of x.

(b) If the median number of goals is 2, find the largest possible value of x.

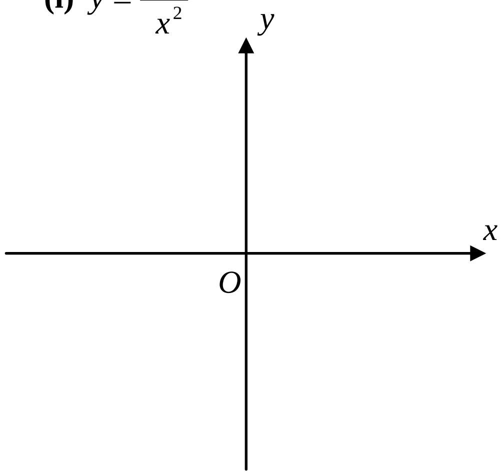
Answer *(a)* ________________ [2]

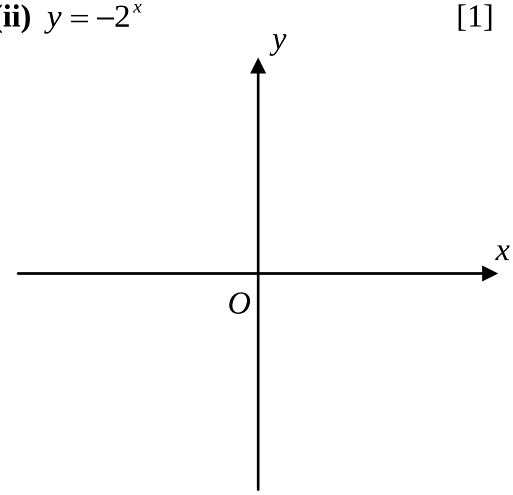
(b) ________________ [2]

For Examiner's Use

19 **(a)** Sketch and label the graph of the following equations.

(i) $y = \dfrac{-1}{x^2}$ [1] **(ii)** $y = -2^x$ [1]



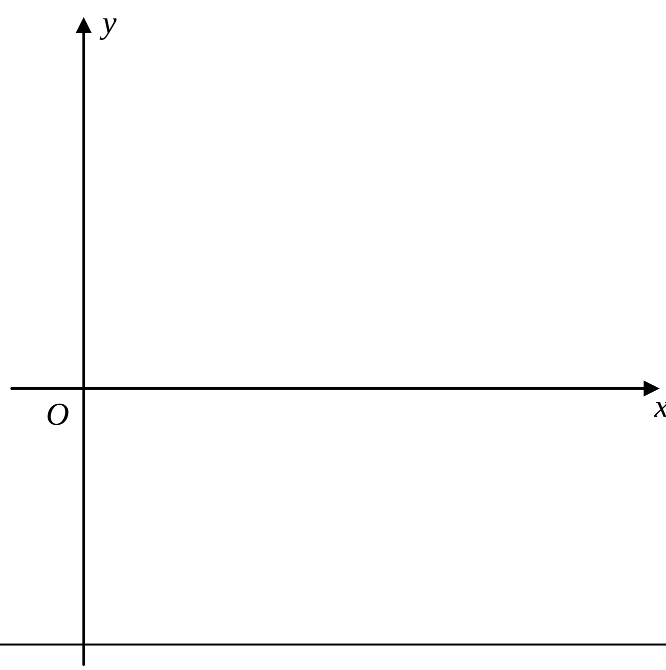


(b) **(i)** Express $y = x^2 - 6x + 5$ as $y = (x+k)^2 + h$.

Answer (b) (i) ____________________ [2]

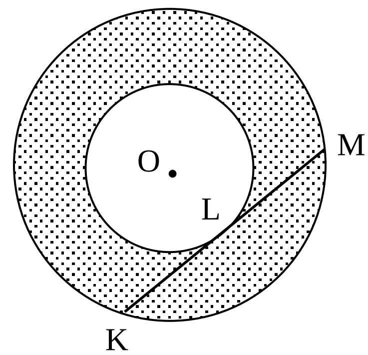
(ii) Hence, sketch the graph of $y = x^2 - 6x + 5$, indicating the coordinates of the turning point, the y-intercept, x-intercepts and the line of symmetry.

Answer (ii) [3]



For Examiner's Use

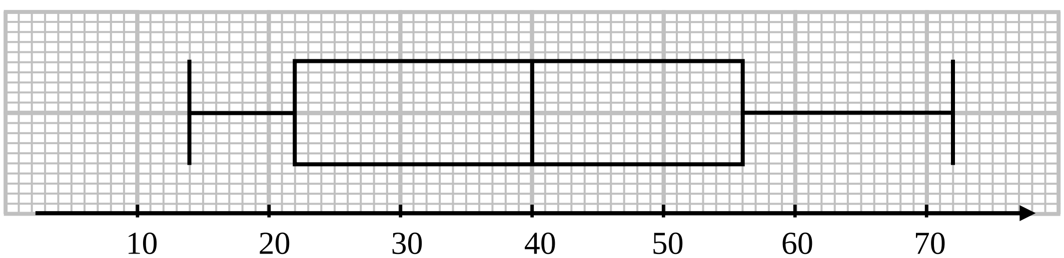
20 The figure below shows two concentric circles with centre at O. Chord $KM = 16\ cm$ on the larger circle is a tangent to the smaller circle at point L. Find the area of the shaded region, giving your answer in terms of π.



Answer _______________ cm^2 [2]

---

21 The box-and-whisker diagram below shows the number of hours per week of work done by part-timers. State the range, median and the interquartile range.



Answer Range = __________ hours [1]

Median = __________ hours [1]

Interquartile range = __________ hours [1]

For Examiner's Use

22 There are 30 students in a class. 14 students are in the NCC and 20 students are in the Soccer team. 4 students are neither members of the NCC nor the Soccer team.

Let ε = {Students in the class}
N = {Students in the NCC}
S = {Students in the Soccer Team}

(a) Draw a Venn Diagram to illustrate the above information. Show on the Venn Diagram the number of elements in each distinct region.

It is also given that C = {Chinese students in the class}
M = {Malay students in the class}
I = {Indian students in the class}

(b) **(i)** Describe in words the meaning of the set notation $M \cap S \neq \{\ \}$.

(ii) Describe what you can deduce from the set notation $I \subset N$.

(iii) Express in set notation:
{Chinese students who are neither in NCC nor the Soccer team}.

Answer (a) [3]

Answer (b) (i) ______________________________

______________________________ [1]

(ii) ______________________________

______________________________ [1]

(iii) ______________ [1]

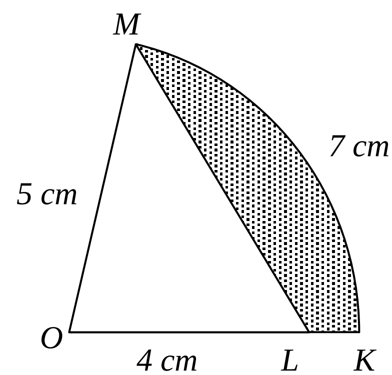
For Examiner's Use

23 KOM is a sector of radius 5 cm and arc length 7 cm. L is a point on OK such that $OL = 4$ cm. Find

(a) $\angle KOM$ in radians,

(b) the perimeter of the shaded region,

(c) the area of the shaded region.



(a) ________________ rad [1]

(b) ________________ cm [2]

(c) ________________ cm^2 [1]

End of Paper

Solutions to Mathematics 4016/01

1 143000 [B1]

2 $m = \dfrac{k}{n^2}$, where k is a constant. Let m_1 and n_1 be the original values $\Rightarrow k = m_1 n_1^2$

If m is increased by 1500%, then $m_2 = 16\,m_1$. [M1]

$\Rightarrow n_2^2 = \dfrac{k}{m_2}$ $\Rightarrow n_2^2 = \dfrac{k}{16m_1}$ $\Rightarrow n_2^2 = \dfrac{m_1 n_1^2}{16m_1}$ $\Rightarrow n_2^2 = \left(\dfrac{n_1}{4}\right)^2$ [M1]

$\Rightarrow n_2 = \dfrac{n_1}{4}$ $\Rightarrow n$ is decreased by 75%. [A1]

3 (a) $\sqrt[3]{z^{-6}} = z^{\frac{-6}{3}} = \dfrac{1}{z^2}$ [B1]

(b) $\left(pq^2\right)^{-5} \times \dfrac{q^0}{p^3} \div \left(p^{-3}q\right)^3 = p^{-5}q^{-10} \times p^{-3} = \dfrac{1}{p^8 q^{10}}$ [B2]

4 (a) Pens (130 - 4) = 126 = $2\times3^2\times7$

Rulers 90 = $2\times3^2\times5$

Erasers (55 - 1) = 54 = $2\times3^2\times3$ [M1]

$\Rightarrow$ She packed $2\times3^2 = 18$ (H.C.F.) pencil boxes. [A1]

(b) She need to buy (7 – 4) = 3 Pens, (5 – 0) = 5 Rulers and (3 – 1) = 2 Erasers.[B2]

5 (a) 1 mark for all 3 answers correct [B1]

Distance (metres)	400	1400	400	200
Time (seconds)	100	400	100	**40**
Speed (m/s)	4.0	**3.5**	4.0	5.0

(b) Total distance = (400 + 1400 + 400 + 200) = 2400 m (total distance-given)

Total time = (100 + 400 + 100 + 40) = 640 seconds [M1]

Average speed = 2400 / 640 = 3.75 m/s [A1]

6 (a) 0.1496 Tm = 0.1496 x 10^{12} $\div 10^3$ km = 1.496×10^8 km [B1]

(b) Time taken = 3.336 ns/m x 0.1496 Tm = (3.336 x 10^{-9}) x (0.1496 x 10^{12}) s

= 499 s [B1]

7 (a) 4 cm^2: 1 km^2 $\Rightarrow$ 2 cm: 1 km $\Rightarrow$ 2 cm: 100 000 cm
$\Rightarrow$ 1 cm: 50 000 cm $\Rightarrow$ 1: 50 000 $\Rightarrow$ n = 50 000 [B1]

(b) For the new map, 4/9 cm^2: 1 km^2 $\Rightarrow$ 2/3 cm: 1 km
$\Rightarrow$ 1 cm: 1.5 km $\Rightarrow$ 16 cm: 24 km [B1]

8 (a) $a^2 - b^2 = (a-b)(a+b)$ [B1]

(b) $\sqrt{8.2^2 - 1.8^2} = \sqrt{(8.2-1.8)(8.2+1.8)} = \sqrt{(6.4)(10)} = \sqrt{64} = 8$ (c.a.o.) [B1]

9 (a) Term 4, $w_4 = 3 \times 2^3 = 24$, Term 5, $w_5 = 3 \times 2^4 = 48$ [B1]

(b) Term n, $w_n = 3 \times 2^{n-1}$ [B1]

10 $\frac{3}{3x-2} - \frac{13}{6x^2+5x-6} = \frac{3}{3x-2} - \frac{13}{(3x-2)(2x+3)}$ [M1]

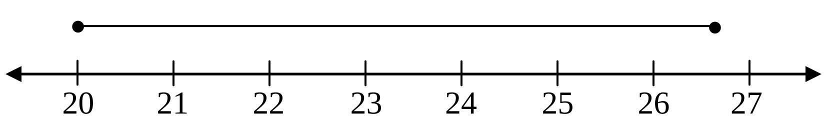
$= \frac{3(2x+3)}{(3x-2)(2x+3)} - \frac{13}{(3x-2)(2x+3)}$

$= \frac{6x+9-13}{(3x-2)(2x+3)} = \frac{6x-4}{(3x-2)(2x+3)}$ [M1]

$= \frac{2(3x-2)}{(3x-2)(2x+3)} = \frac{2}{(2x+3)}$ [A1]

11 (a) $x \geq 20$ [B1]
$90x + 60x \leq 4000$ [B1]

(b) $x \geq 20$ and $150x \leq 4000$ $\Rightarrow$ $20 \leq x \leq 26\frac{2}{3}$ [B1]



[B1]

(c) Maximum sets = 26 [B1]

12 (a) Gradient of line $L_1 = -\frac{1}{2}$ $\Rightarrow$ $y = -\frac{1}{2}x + c$ [M1]

At point $(6\ ,\ -8)$, $-8 = -\frac{1}{2}(6) + c$ $\Rightarrow$ c = -5

The equation of line L_2 is $y = -\frac{1}{2}x - 5$ OR $2y + x = -10$ [A1]

(b) Distance between $(6\ ,\ -8)$ & $(-2\ ,\ -2) = \sqrt{(6-(-2))^2 + (-8-(-2))^2}$ = 10 units [B1]

13 [M1]

Sum	1	2	3	4
1	-	3	4	5
2	3	-	5	6
3	4	5	-	7
4	5	6	7	-

P (sum of both numbers is odd) = $\frac{8}{12} = \frac{2}{3}$ [A1]

14 (a) $A = \begin{pmatrix} 200 & 600 \\ 300 & 900 \end{pmatrix}$, $B = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$, [M1]

$AB = \begin{pmatrix} 200 & 600 \\ 300 & 900 \end{pmatrix}\begin{pmatrix} 10 \\ 5 \end{pmatrix} = \begin{pmatrix} 5000 \\ 7500 \end{pmatrix}$ [A1]

(b) $C = \begin{pmatrix} 8 \\ 4.5 \end{pmatrix}$ [M1]

$AC = \begin{pmatrix} 200 & 600 \\ 300 & 900 \end{pmatrix}\begin{pmatrix} 8 \\ 4.5 \end{pmatrix} = \begin{pmatrix} 4300 \\ 6450 \end{pmatrix}$ [A1]

(c) $D = \begin{pmatrix} 5000 \\ 7500 \end{pmatrix} - \begin{pmatrix} 4300 \\ 6450 \end{pmatrix} = \begin{pmatrix} 700 \\ 1050 \end{pmatrix}$ [B1]

The elements of *D* represent the amount of discount to be given on a working day and a non-working day. [B1]

15 (a) Deduct 1 mark for each mistake. [B2]

Mathematics Test Marks							
3	5						
5	0						
6	0	1	2	2	5	5	7
7	0	1	3	6	7		
8	1	2	3				
9	1						

Key 3 | 5 means 35 marks

(b) Mean marks scored =

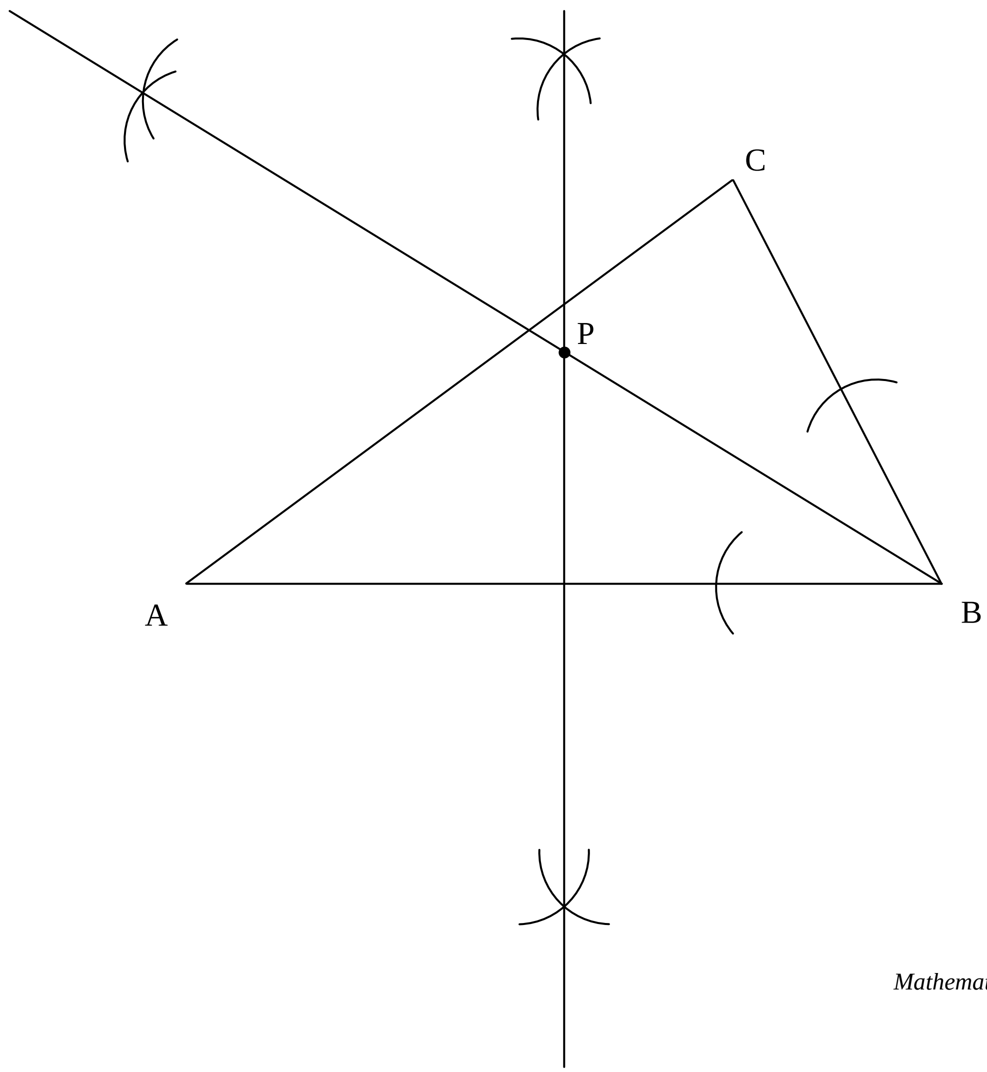
$$\frac{35+50+60+61+62+62+65+65+67+70+71+73+76+77+81+82+83+91}{18}$$

$= \dfrac{1231}{18} = 68.38 = 68.4$ (correct to 3 sig. fig.) [B1]

(c) Median marks scored = $\dfrac{67+70}{2} = 68.5$ [B1]

(d) Modal marks scored = 62 **and** 65 (no marks if only 1 ans given) [B1]

16 Correct ΔABC drawn. [B1]
Angle bisector of $\angle ABC$. [B1]
Perpendicular bisector of line AB . [B1]
Locate point *P* as the intersection of the two bisectors. [B1]



17 (a) 6-mth Road Tax on 1,500 cc = [$250 + 0.375 $\times$ (1,500 - 1,000)] $\times$ 0.782 [M1]

= $342.125 = $342.13 [A1]

(b) $1,446.70 is in the category of 1,600 < EC $\leq$ 3,000

$\Rightarrow$ [$475 + 0.75 $\times$ (EC - 1,600)] $\times$ 0.782 = 1,446.70 $\div$ 2 [M1]

$\Rightarrow$ [$475 + 0.75 $\times$ (EC - 1,600)] = 925

$\Rightarrow$ (EC - 1,600) = 600 $\Rightarrow$ EC = 2,200 [A1]

18 (a) Mean = (0x7 + 1$\times x$ + 2x5 + 3x3 + 4x1 + 5x4) / (7+ x +5+3+1+4) > 2 [M1]

$\Rightarrow$ (43 + x) > (20 + x) (2) $\Rightarrow$ 3 > x

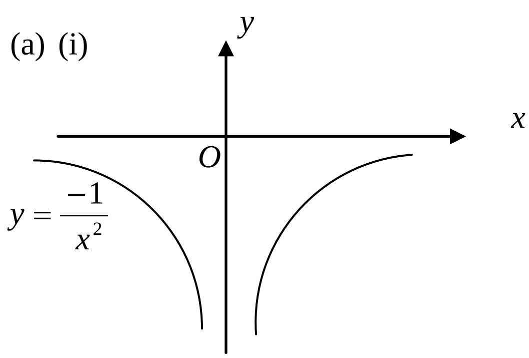
$\Rightarrow$ Largest value of x is 2. [A1]

(b) If x = 5 $\Rightarrow$ median is at position (25 + 1)/2 = 13 Median = 2

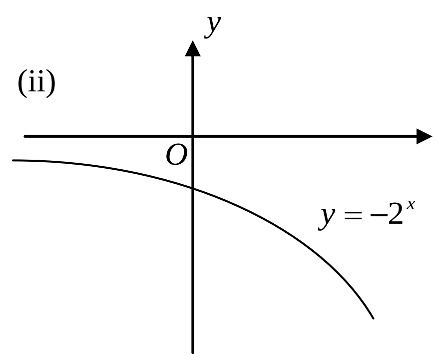
If x = 6 $\Rightarrow$ median is at position (26 + 1)/2 = 13.5 Median = 1.5

$\Rightarrow$ the largest possible value of x = 5 [B2]

19 (a) (i) [B1]



(ii) [B1]



(b) (i) $y = 5 - 6x + x^2$ compared with $y = (x+k)^2 + h = x^2 + 2kx + (k^2 + h)$

By comparing the coefficients, we get

$-6 = 2k \Rightarrow k = -3$ and $(k^2 + h) = 5 \Rightarrow h = -4$ [M1]

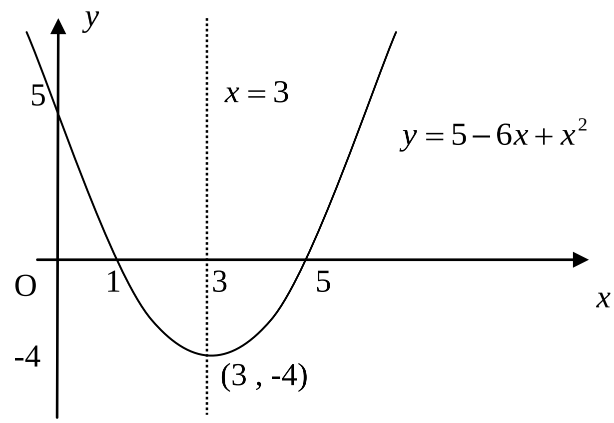
$\Rightarrow$ $y = 5 - 6x + x^2$ can be expressed as $y = (x-3)^2 - 4$ [A1]

(ii) At the y-axis, $x = 0$ $\Rightarrow$ $y = 5$

At the x-axis, $y = 0$ $\Rightarrow$ $x - 3 = 2$ or -2 $\Rightarrow$ $x = 5$ or $x = 1$

Line of symmetry is $x = 3$

The minimum value of y occurs when $x = 3$, $y = -4$, i.e. at $(3\,,\,-4)$



2 marks (sketch)
1 mark (line of symmetry)
Or 1 mark (coordinates of turning pt)

20 Let the radius of the smaller circle OL be r & the radius of the larger circle OM be R

By Pythagoras' Theorem, $r^2 + 8^2 = R^2 \quad \Rightarrow \quad R^2 - r^2 = 8^2$ [M1]

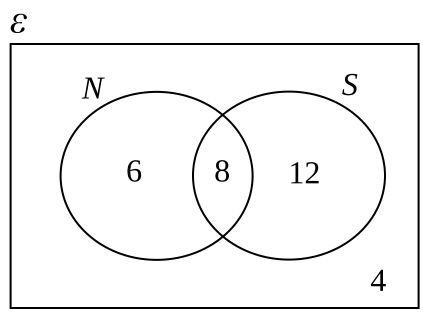
$\Rightarrow$ Area of shaded region = $\pi R^2 - \pi r^2 = \pi\left(R^2 - r^2\right) = 64\pi$ [A1]

21 Range = 58 hrs [B1]

Median = 40 hrs [B1]

Interquartile range = 34 hrs [B1]

22 (a)



Deduct 1 mark each for two overlapping circles N & S missing/wrong, [B1]

Deduct 1 mark each for missing/wrong label of ε, N & S, [B1]

Deduct 1 mark each for any numbers (6, 8, 12, 4) missing/wrong. [B1]

(b) (i) $M \cap S \neq \{\ \}$ means there are Malay students in the soccer team. [B1]

(ii) Since I is a proper subset of N, therefore all Indian students are members of the NCC [B1]

(iii) $C \cap (N \cup S)'$ OR $C \cap (N' \cap S')$ [B1]

23 (a) Arc Length = $r\theta \quad \Rightarrow \quad \angle KOM = \frac{7}{5}$ rad = 1.4 rad [B1]

(b) $ML^2 = 5^2 + 4^2 - 2(5)(4)\cos(1.4) \quad \Rightarrow \quad ML = 5.848$ cm

$\Rightarrow$ perimeter = 5.848 + (5 - 4) + 7 = 13.8 cm (3 s.f.) [B2]

(c) Area of shaded region = $\frac{1}{2}r^2\theta$ - $\frac{1}{2}ab\sin C$ = 17.5 – 9.85 = 7.65 cm^2 [B1]