Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = $\pi r l$

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

This paper consists of 29 printed pages.

1.

Country	Population
China	1.39 billion
Singapore	5 399 200

Using the information from the table above,

(a) express the population of Singapore in standard form.

Answer (a) ..... [1]

(b) express, in the form $1:n$, the ratio
population of Singapore: population of China.

Answer (b) 1: ..... [1]

2. (i) Solve the inequalities $-20 < 16 + 4(2x - 3) \leq 20$.

Answer (i) ..... [2]

(ii) Represent the solution on the number line below

Answer

[1]

3. The diameter of the nucleus of a Hydrogen atom is about 1.75×10^{-6} nanometres.
Express 1.75×10^{-6} nanometres in metres, giving your answer in standard form.

Answer ..... m [1]

4. Factorise fully

(a) $x^2y - 4y^3$,

Answer (a) ..... [2]

(b) $2a - 4b - 6ab + 3a^2$.

Answer (b) ..... [2]

5. The scale of a map is 1 : 250000.

(a) Calculate the distance, in km, between 2 towns which are 9.2 cm apart on the map.

Answer (a) ..... km [1]

(b) Calculate the area in cm^2 on the map which represents a lake of area $25km^2$.

Answer (b) ..... cm^2 [2]

6. Solve the simultaneous equations

$$x + 2y = 2$$
$$8y - 2x = -32$$

Answer $x =$ .....

$y =$ ..... [3]

7. At noon, the temperature at the foot of Mount Fuji was 25.9 °C while the temperature at the summit of the mountain was −5.1 °C.

(a) What is the difference in temperature between the foot and the summit of Mount Fuji at noon?

Answer (a) ..... °C [1]

Given that the height of Mount Fuji is 3776 m and the temperature changed uniformly with height,

(b) find the height at which the temperature was 0 °C at noon.

Answer (b) ..... m [1]

8. Mr Lim invested \$25 000 in a bank that pays 1.68% compound interest per annum compounded twice yearly. Calculate the total amount he has in the bank after 5 years.

Answer \$ ..... [2]

9. The length and breadth of a rectangle is increased by 60%. If the original dimension of the rectangle was 8 cm by 3 cm, find the percentage increase of the area of the final rectangle.

Answer ..... [2]

10. One of the interior angles of a polygon is 52^o. The remaining interior angles are 151^o each. Find the number of sides of the polygon.

Answer ..... [2]

11. $e = \{\text{Integers from 1 to 10}\}$

$A = \{\text{Multiples of 2}\}$

$B = \{\text{Multiples of 3}\}$

$C = \{\text{Perfect Squares}\}$

Draw a Venn diagram to illustrate the information. Hence, or otherwise, find $n(A \cup B \cup C)'$.

[2]

12. Simplify $\frac{5x - 32}{2x^2 + 3x - 20} + \frac{3}{2x - 5}$.

Answer ..... [3]

13. Written as the product of its prime factors,

$$75460 = 2^2 \times 5 \times 7^3 \times 11$$

(a) Find the smallest positive integer k such that $75460k$ is a perfect square.

Answer (a) ..... [1]

(b) Write 1320 as the product of its prime factors.

Answer (b) ..... [1]

Hence write down

(i) the LCM of 1320 and 75460, giving your answer as the product of its prime factors,

Answer (b)(i) ..... [1]

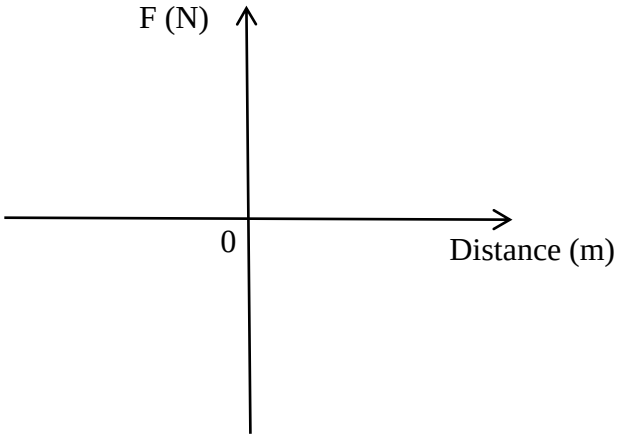
(ii) the greatest number that will divide both 75460 and 1320 exactly.

Answer (b)(ii) ..... [1]

14. The force of attraction between two objects (F Newton) is inversely proportional to the square of the distance (*d* metres) between the two objects.

(a) Draw, on the axes in the answer space, the graph representing the relationship between the force of attraction and the distance.

Answer (a)



[1]

(b) When the distance is *d* metres, the force of attraction of two objects is 155N. If the distance is increased by 50%, find

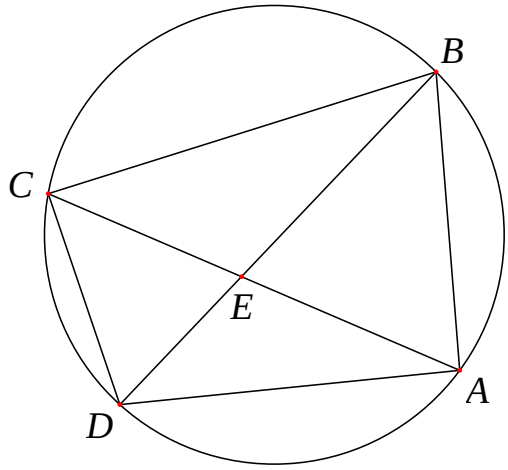
(i) an expression for the distance,

Answer (b)(i) ..... m [1]

(ii) the new force of attraction.

Answer (b)(ii) ..... N [1]

15. In the diagram, the points A, B, C and D lie on the circumference of the circle. The diagonals of the quadrilateral, AC and BD, intersect at E.



(a) Show that triangle ABE is similar to triangle DCE.

Answer (a) .....

.....

.....

..... [2]

(b) If AB is parallel to CD and area of triangle ABE: area of triangle DCE is 9: 4.

Find the value of

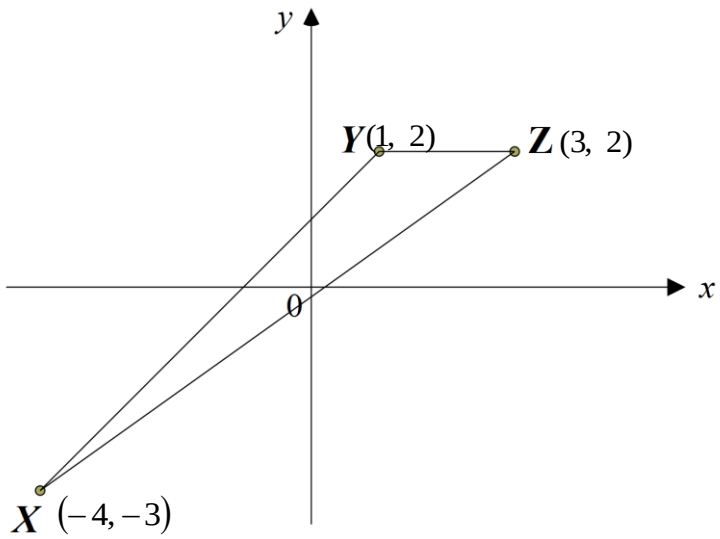
(i) $\dfrac{\text{area of } \Delta AED}{\text{area of } \Delta DEC}$,

Answer (b)(i) ..... [1]

(ii) $\dfrac{\text{area of } \Delta DEC}{\text{area of quadrilateral } ABCD}$.

Answer (b)(ii) ..... [1]

16. The points $X\ (-4, -3)$, $Y\ (1,\ 2)$ and $Z\ (3,\ 2)$ are shown in the diagram.



Find

(a) the gradient of the line *XZ*,

Answer (a) ..... [1]

(b) the equation of the line which passes through $(3,\ 7)$ and is parallel to *XY*.

Answer (b) ..... [1]

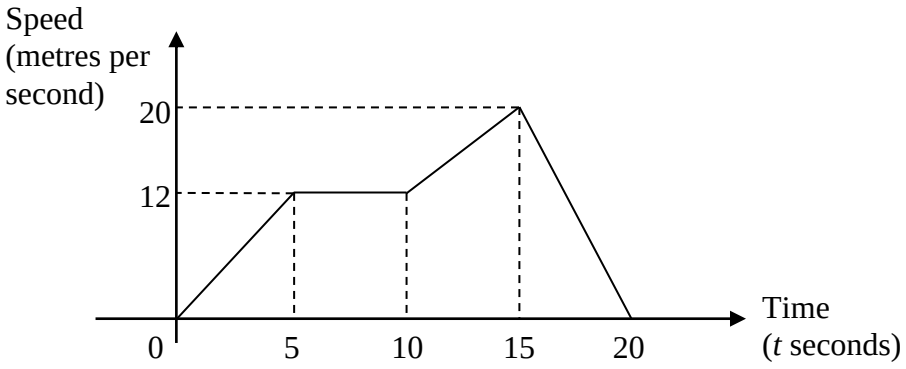
(c) the length of *XY*,

Answer (c) ..... units [1]

(d) the exact value of sin $\angle$ *XYZ*.

Answer (d) ..... [1]

17. The diagram shows the speed-time graph of a car travelling a total distance of 220m in 20 s.



Find

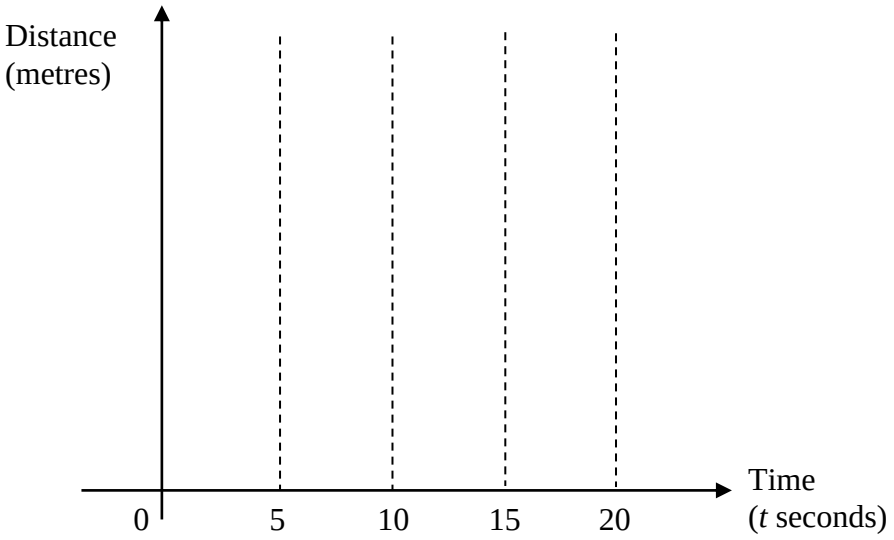
(i) the retardation of the car at t = 19s,

Answer (a)(i) ..... m/s^2 [1]

(ii) the time taken for the particle to travel 54m.

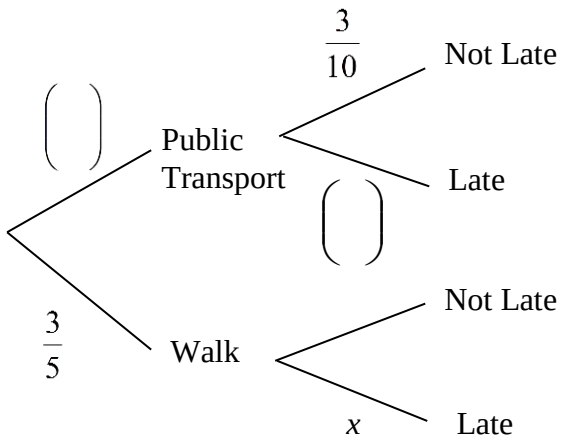
Answer (a)(ii) ..... s [3]

(b) On the given axes, complete the sketch of the distance-time graph for the whole journey.



[3]

18. A survey was conducted on a group of students on how they get to school this morning. 40% of the students were late and 60% indicated walking to school. 30% of those who took public transport were not late. Complete the following tree diagram.



[1]

(a) Show that the value of x is $\frac{1}{5}$.

Answer (a) .....

.....

.....

..... [1]

(b) A student was chosen at random, find the probability that the chosen person is

(i) a someone who took public transport and was late

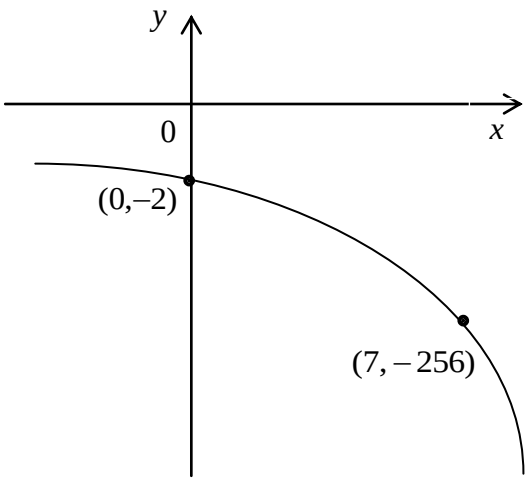
Answer (i) ..... [1]

(ii) either someone who took public transport and was not late or someone who walks and was late.

Answer (ii) ..... [1]

19. (a) The sketch shows the graph of $y = ab^x$. The curve passes through the points $(0,-2)$ and $(7,-256)$.

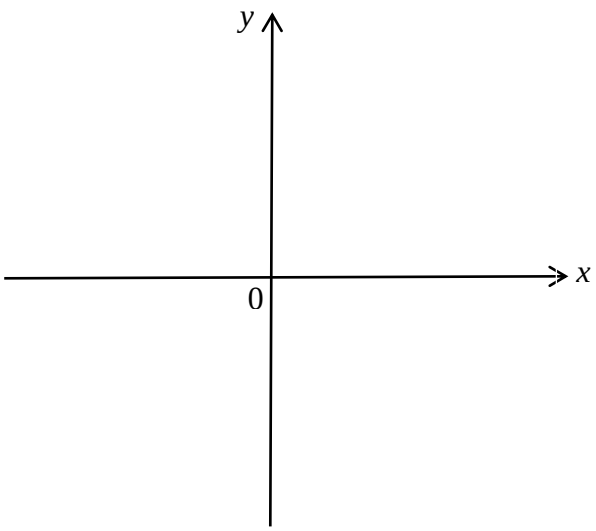
Find the values of *a* and *b*.



Answer $a =$ ..... [1]

$b =$ ..... [1]

(b) Sketch the graph of $y = 4(4 - x^2)$.



[2]

20. (a) The table below gives the life span of 80 light bulbs produced by Factory *A*.

Life Span (x hours)	Number of light bulbs
$0 < x \leq 10$	3
$10 < x \leq 20$	10
$20 < x \leq 30$	27
$30 < x \leq 40$	31
$40 < x \leq 50$	9

Calculate an estimate for

(i) the mean life span of the light bulbs,

Answer (a)(i) ..... h [1]

(ii) the standard deviation of the life span of the light bulbs.

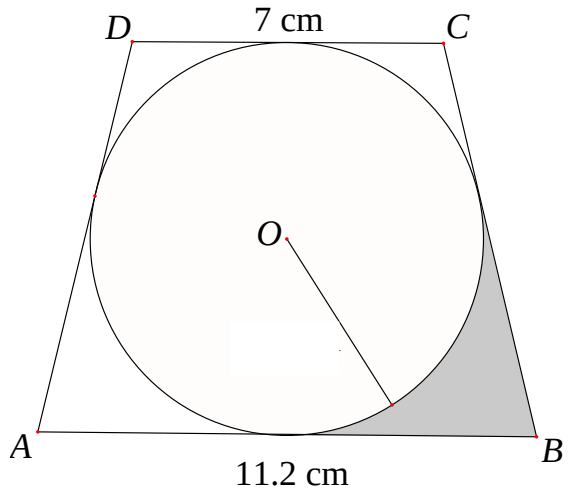
Answer (a)(i) ..... h [2]

(b) The mean of the life span of 80 light bulbs from Factory *B* is x hours.

If the life span of each light bulb is increased by 3 hours now, write down in terms of x, the mean,

Answer (b) ..... h [2]

21. *ABCD* is a quadrilateral. *AB* = 11.2 cm, *DC* = 7 cm and *AD* = *BC*. A circle, centre *O*, is inscribed in the quadrilateral.



(a) Write down the length of *BC*.

Answer *(a)* ..... cm [1]

(b) Show that angle *ABC* = 1.34 rad.

Answer

[1]

(c) Calculate the radius of the circle

Answer *(c)* ..... cm [1]

(d) Calculate the area of the shaded region.

Answer *(c)* ..... cm^2 [3]

22. The marks of 20 pupils for a Mathematics Competition are represented in a stem and leaf diagram.

Stem	Leaf						
4	2	7					
5	1	3	6	6	6	8	9
6	1	4	4	5			
7	0	2	4	7			
8	3	3	9				

Key: 6 | 4 represents 64 marks

(a) Find the
(i) median,
(ii) mean.

Answer *(a)(i)* ..... [1]

(a)(ii) ..... [1]

(b) The number of pupils from each stem is represented as a category in a pie chart. Calculate the angle of the sector representing pupils with more than 75 marks.

Answer *(b)* ..... ° [1]

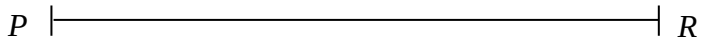
(c) One of the marks was deleted at random; find the new median if the mean mark of the remaining 19 pupils is 63.

Answer *(c)* ..... [2]

23. (a) Construct a rhombus *PQRS* in which *QS* = 6cm.
PR has already been drawn.

[2]

Answer (a), (b) and (c).



(b) Construct
(i) the angle bisector of angle *PRS*, [1]
(ii) the perpendicular bisector of *PQ*. [1]

(c) Mark clearly a possible point which is inside the rhombus, equidistant from *P* and *Q*, and is nearer to *SR* than *PR*.
Label this point *X*. [1]

~ End of Paper 1~

Answer **all** the questions.

1. (a) Simplify $12x^2y \times \frac{3x^{-5}}{8x^3y^{-3}}$. [2]

   (b) Solve the equation $\left(\frac{1}{4}\right)^{-2x} = \sqrt[3]{2^{(5x+6)}}$. [3]

   (c) Given that $3m^2 = \sqrt{\frac{x+y}{2xy}}$, express x in terms of m and y. [3]

2. (a) An electronic shop sells 2 types of portable MP3 player, Brand A and Brand B. The cost price of Brand A is \$63.50, and the cost price of Brand B is $\frac{3}{5}$ that of Brand A.

   (i) The shopkeeper decides to mark up the price of the players to make a profit of 20%. The GST of 7% is then added to give the selling price. What is the selling price of Brand A player? Give your answer to the nearest dollar. [2]

   (ii) During the Great Singapore Sale, Brand B player was sold for \$55.30 after 30% discount. What was the price before the discount? [1]

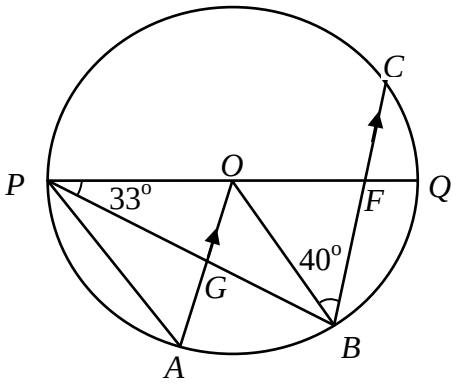
   (iii) What is the percentage profit for Brand B player during the Great Singapore Sale? [2]

   (b) Peter decided to buy a car priced at \$128 340. He paid \$52 000 in cash as down payment and borrowed the remainder from a finance company for a period of 10 years. If the finance company charges a simple interest rate of 1.85% per annum, calculate

   (i) the total interest paid by Peter, [2]

   (ii) the amount of each monthly instalment, giving your answer to the nearest dollar. [2]

3.



In the diagram above, *POQ* is the diameter of the circle with centre *O*.
Given that points *A, B, C, P* and *Q* are points on the circumference of the circle, *CB* is parallel to *OA*, ∠FBO = 40° and ∠BPQ = 33° , calculate

(a) ∠BOQ, [1]
(b) ∠QFC, [2]
(c) ∠BPA, [1]
(d) ∠OGB. [2]

4. A water tank holds 6 m^3 of water when it is full. A hot water tap supplies water at a rate of x m^3 per minute and a cold water tap supplies water at a rate of $(20 - \frac{36}{x})$ m^3 per minute. Filling the tank using cold water will take 4 minutes longer than filling the tank using hot water.

(a) Form an equation in x and show that it reduces to $37x^2 - 12x - 108 = 0$. [5]
(b) Hence, or otherwise, solve $37x^2 - 12x - 108 = 0$, giving your answers correct to 2 decimal places. [3]
(c) Find, to the nearest minute, the time taken to fill the tank using the hot water tap. [2]

5. (a) The object shown in Diagram I is formed when a smaller right hollow cone is cut off from a large one. The height of the object is x cm, the height and the base radius of the large cone are 30 cm and 24 cm respectively.

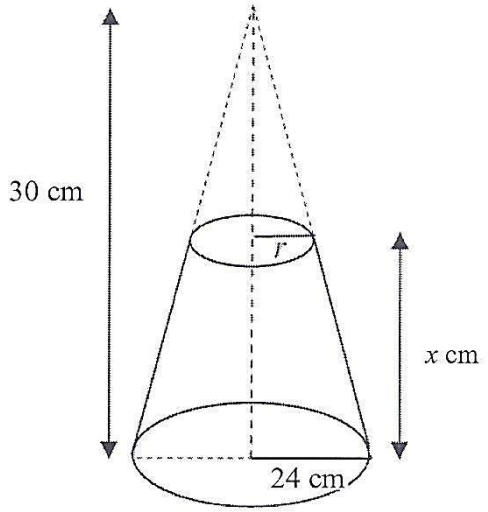


Diagram I

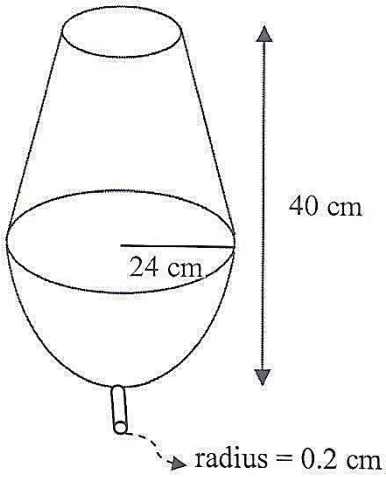


Diagram II

(i) Show that the radius of the small cone, r, in terms of x, is $\frac{4}{5}(30 - x)$ cm. [2]

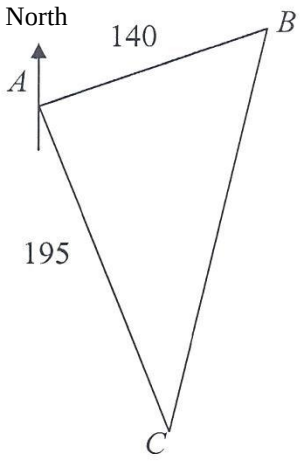
(ii) Hence show that the volume of the object is $\frac{16\pi}{75}[27000 - (30 - x)^3]$ cm^3.[3]

(b) As shown in Diagram II, a manufacturer decided to fix a hollow hemisphere of radius 24 cm to the object in (a) to form an open container. The height of the container is 40 cm and its thickness is negligible.

(i) Calculate, in litres, the amount of water needed to fill the container to its rim. [4]

(ii) The water is then allowed to escape through a cylindrical pipe of radius 0.2 cm at the bottom of the hemisphere at a uniform rate of 10 cm/s. Calculate, to the nearest second, the time taken for all the water to escape. [3]

6.



A, B and C are points on level ground, where the bearing of C from A is 165° and the bearing of A from B is 245°. $AB = 140$ m and $AC = 195$ m.

(a) Calculate

(i) $\angle BAC$, [2]

(ii) the length of BC, [2]

(iii)the bearing of C from B. [3]

(b) Mike walked from B to a point nearest to A along BC. How far is she from A? [2]

(c) A flagpole stands at A. Given that the greatest possible angle of elevation from a point along BC to the top of the flagpole is 17°, find the height of the flagpole. [2]

7. A shop sold 3 different brands of sports shirts, known as Brand A, Brand B and Brand C. The table below shows the stocks for the first 3 days of a week.

	Day 1	Day 2	Day 3
Brand A	32	23	18
Brand B	12	26	24
Brand C	21	19	31

(a) It is given that $S = \begin{pmatrix} 32 & 23 & 18 \\ 12 & 26 & 24 \\ 21 & 19 & 31 \end{pmatrix}$ and that $X = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

(i) Evaluate **SX**. [1]

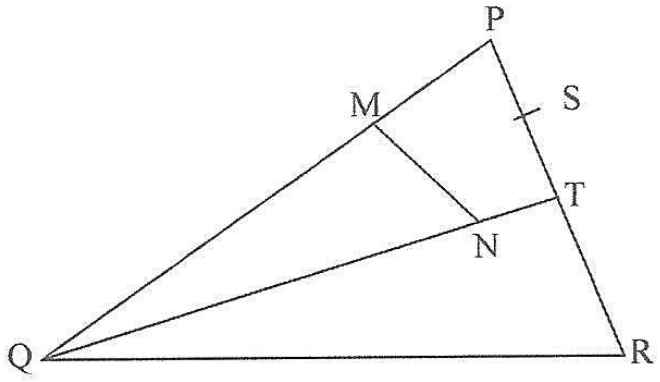
(ii) What do the numbers in **SX** represent? [1]

For the first three days of the week, the shop owner sold only 20% of the Brand A shirts, 50% of the Brand B shirts and 40% of the Brand C shirts.

(b) If Brand A, Brand B and Brand C shirts are sold at \$80, \$65 and \$45 respectively, form three matrices such that their product will give the total amount from the sale of the shirts for the first three days of the week. [3]

(c) Evaluate this product. [2]

8. In the diagram, $PQ = 3PM$ and $PR = 4PS$. T is the mid-point of PR and $TN = \frac{1}{5}TQ$.



(a) Given that $\overrightarrow{PS} = 2\boldsymbol{a}$ and $\overrightarrow{PM} = 2\boldsymbol{b}$, express as simply as possible, in terms of $\boldsymbol{a}$ and/or $\boldsymbol{b}$,

(i) $\overrightarrow{MQ}$, [1]

(ii) $\overrightarrow{MR}$, [1]

(iii) $\overrightarrow{TQ}$, [1]

(iv) $\overrightarrow{MN}$. [2]

(b) Show that M, N and R are collinear. [2]

9. (a) Dots and triangles are used to form the patterns below.

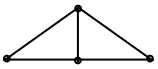


Figure 1

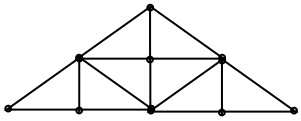


Figure 2

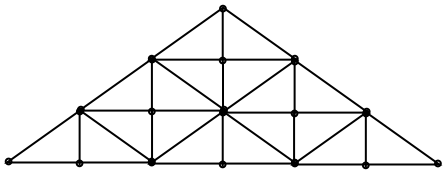


Figure 3

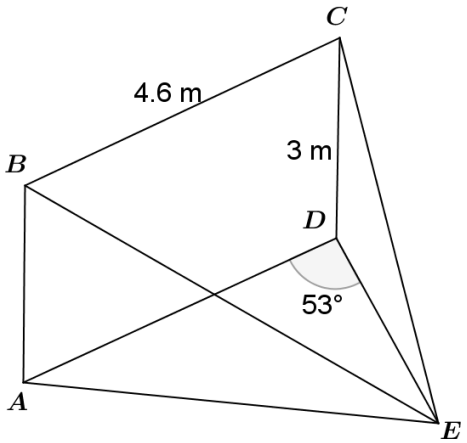
Figure	Total number of dots	Number of small triangles
1	4	2
2	9	8
3	16	18
4	a	b
. . .	. . .	. . .
n	p	q

(i) Find the values of a and b. [2]

(ii) How many small triangles would you expect to see in Figure 14? [1]

(iii) Write down expressions for p and q in terms of n. [2]

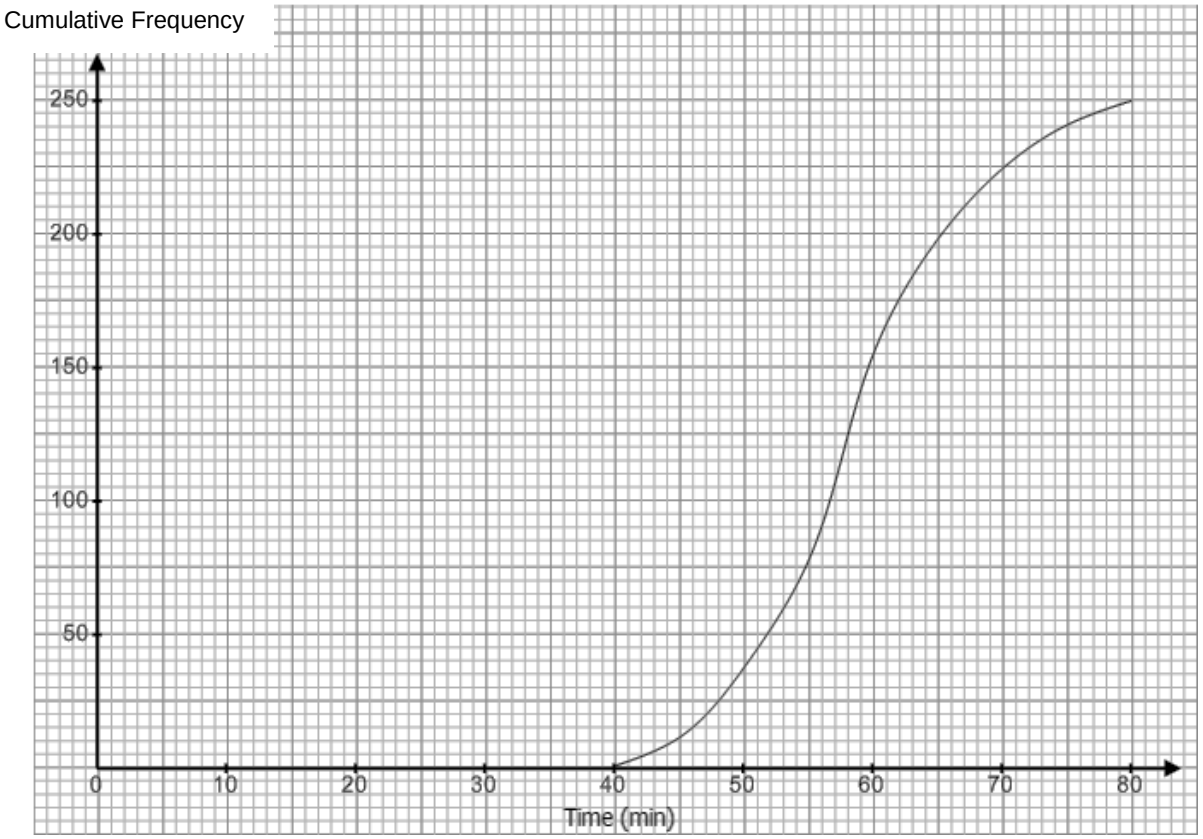
(b)



The diagram shows a vertical rectangular board $ABCD$ with AD on the ground supported by cables BE and CE where E is also on the ground. Given that $CD = 3$ m, $BC = 4.6$ m, $\angle ADE = 53°$ and ΔADE is isosceles with $AE = DE$. Find

(i) DE, [1]
(ii) $\angle CED$, [1]
(iii) volume of $ABCDE$. [3]

10.



The cumulative frequency curve above shows the marks obtained in a Mathematics competition for a group of 250 students from **School X**.

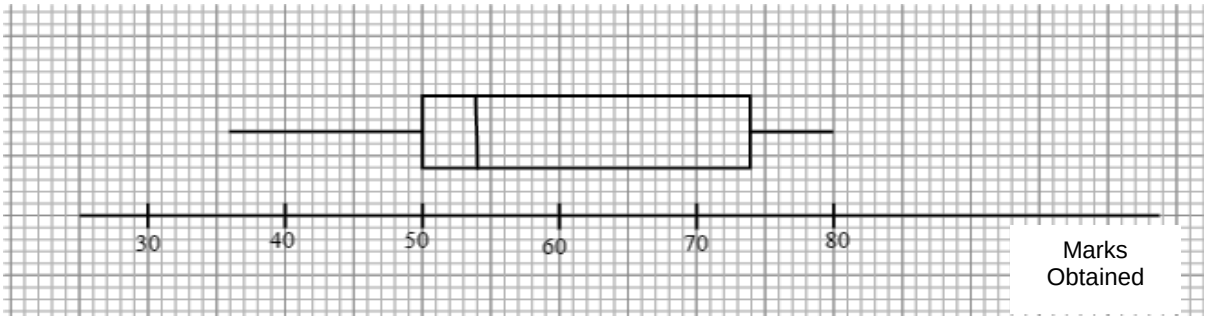
(a) Use the curve to find

(i) the median mark for the Mathematics competition, [1]

(ii) the interquartile range of the distribution, [2]

(iii) the probability that two students selected from the group obtained more than 68 marks in the Mathematics competition. [2]

(b) For the same Mathematics competition, the marks obtained by another 250 students from **School Y** are summarized in the box-and-whisker plot below.
Compare the results of **School X** and **School Y** in two different ways. [2]



11. **Answer the whole of this question on a sheet of graph paper.**

The table below shows some values of x and the corresponding values of y, correct to 1 decimal place, related by the formula, $y = 4.8x - x^2 + 2$.

$\boldsymbol{x}$	0	1	2	3	4	5
$\boldsymbol{y}$	2	5.8	7.6	7.4	5.2	p

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to 1 second, draw a horizontal x-axis for $0 \leq x \leq 5$. Using a scale of 2 cm to 2 m, draw a vertical y-axis for $0 \leq y \leq 8$. On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find

(i) the values of x when $y = 6$, [1]

(ii) the maximum value of the graph. [1]

(d) (i) By drawing a tangent, find the gradient of the graph at $x = 4$. [2]

(ii) Explain what your answer to (d)(i) tells you about the nature of the gradient at $x = 4$. [1]

(e) Another graph $y = x + 1$ is to be added to the existing graph.

(i) On the same axes, draw the graph for $0 \leq x \leq 5$. [1]

(ii) State the value of x when the two graphs intersect. [1]

(iii) This value of x is a solution of the equation $x^2 + Ax + B = 0$.
Find the value of A and of B. [2]

---------- End of Paper 2----------

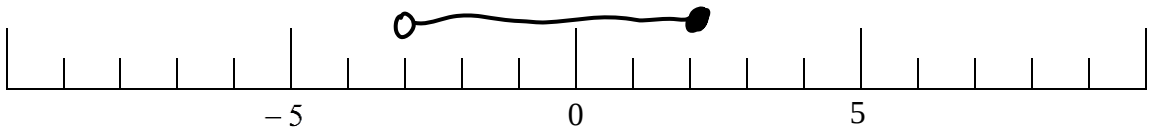
Answer Key

Paper 1

1(a) 5.40×10^{6} (b) 257

2(i) $-3 < x \leq 2$

(ii)



3. 1.75×10^{-15}

4(a) $y(x+2y)(x-2y)$ (b) $(2+3a)(a-2b)$

5(a) 23 (b) 4

6. $x = 6\frac{2}{3}, y = -2\frac{1}{3}$

7(a) $31°C$ (b) 3150 m

8. \$27181.18

9. increase of 156%

10. 9

11. $n(A \cup B \cup C)' = 2$

12. $\frac{4}{x+4}$

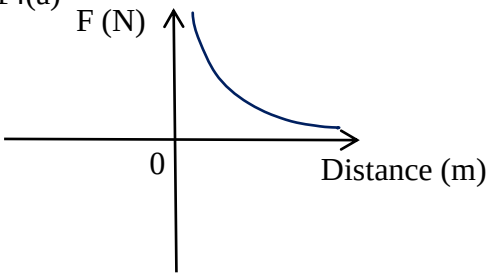
13(a) 385

13(b) $2^3 \times 3 \times 5 \times 11$

13(bi) $2^3 \times 3 \times 5 \times 7^3 \times 11$

13(bii) 220

14(a)



14(bi) $\frac{3}{2}$d

14(bii) 68.9 N

15(bi) $\frac{3}{2}$

15(bii) $\frac{4}{25}$

16(a) $\frac{5}{7}$

16(b) $y = x + 4$

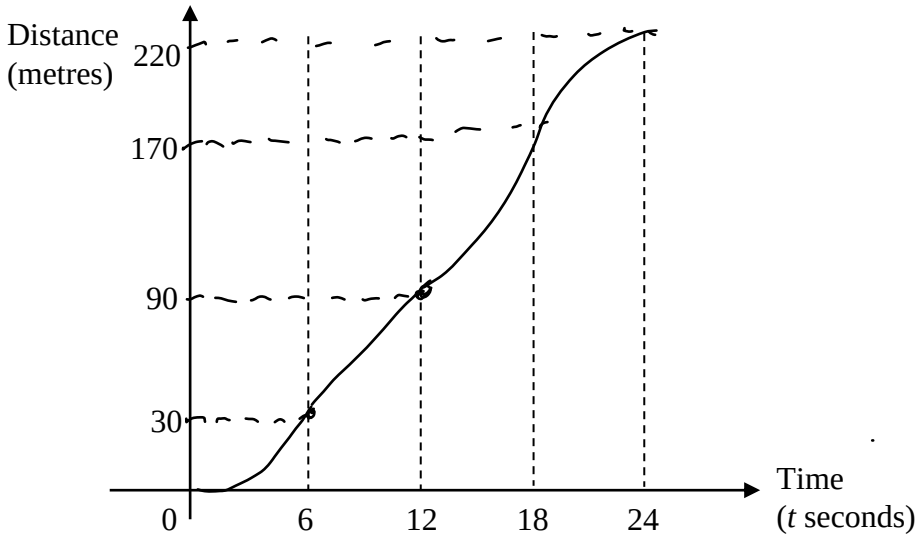
16(c) $\sqrt{50}$

16(d) $-\frac{1}{\sqrt{2}}$

17(ai) 4

17(aii) 7

17(b)



18(bi) $\frac{7}{25}$ 18(bii) $\frac{6}{25}$

19(a) $a = -2, b = 2$

20(ai) 29.1h 20(aii) 9.71 20(b) $x + 3$

21(a) 9.1cm 21(c) 4.43 cm (d) 7.12 cm^2

22(ai) 62.5 22(aii) 64
22(b) 72° 22 (c) 61

23) Construction question

Paper 2

1(a) $\frac{9y^4}{2x^6}$ 1(b) $x = \frac{6}{7}$ 1(c) $x = \frac{y}{18ym^4 - 1}$

2(ai) \$81.53 2(aii) \$79 2(aiii) 45.1 %
2(bi) \$14122.90 2(bii) \$754

3(a) 66° 3(b) 74° 3(c) 20° 3(d) 107°

4(b) 1.88 or -1.55 4(c) 3 min

5(bi) 45.2 *l* 5(bii) 35977 s

6(ai) 100° 6(aii) 259 m 6(aiii) 197.2°
6(b) 104 m 6(c) 31.7 m

7(ai) $\begin{pmatrix} 73 \\ 62 \\ 71 \end{pmatrix}$ 7(aii) Total sales of each brand in these 3 days.

7(b) $\begin{pmatrix} 73 & 62 & 71 \end{pmatrix}\begin{pmatrix} 0.2 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.4 \end{pmatrix}\begin{pmatrix} 80 \\ 65 \\ 45 \end{pmatrix}$

7(c) $\begin{pmatrix} 4461 \end{pmatrix}$

8(ai) $4\mathbf{b}$ 8(aii) $2(4\mathbf{a} - \mathbf{b})$ 8(aiii) $2(3\mathbf{b} - 2\mathbf{a})$ 8(aiv) $\frac{4}{5}(4\mathbf{a} - \mathbf{b})$

9(a)(i) a=25, b=32 9(aii) 392 9(aiii) $p = (n+1)^2,\ q = 2n^2$
9(bi) 3.82 m 9(bii) 38.1° 9(biii) 14.0 m^3

10(ai) 58 10(aii) 10 10(aiii) $\frac{119}{6225}$

11(a) $p = 1$

11(ci) $x = 1.1$ and 3.7

11(cii) max. $y = 7.76$

11(di) -3.22

11(ei) $4.05(\pm 0.1)$

11(eiii) $A = -3.8, B = -1$