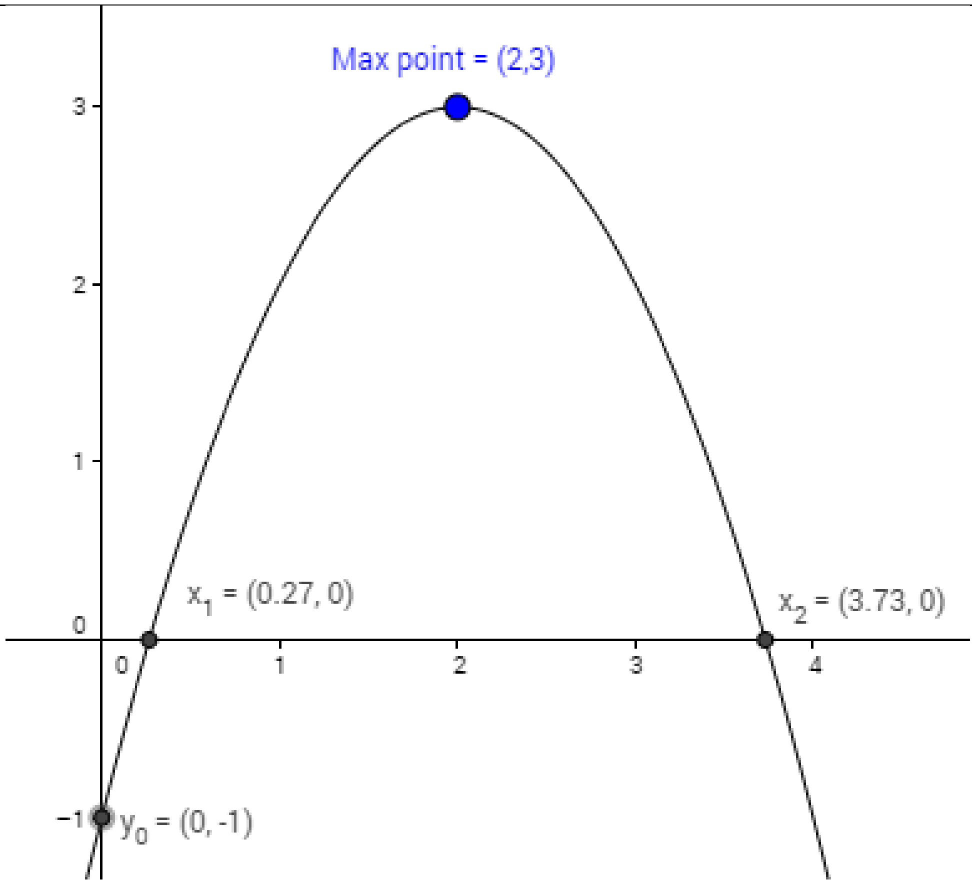
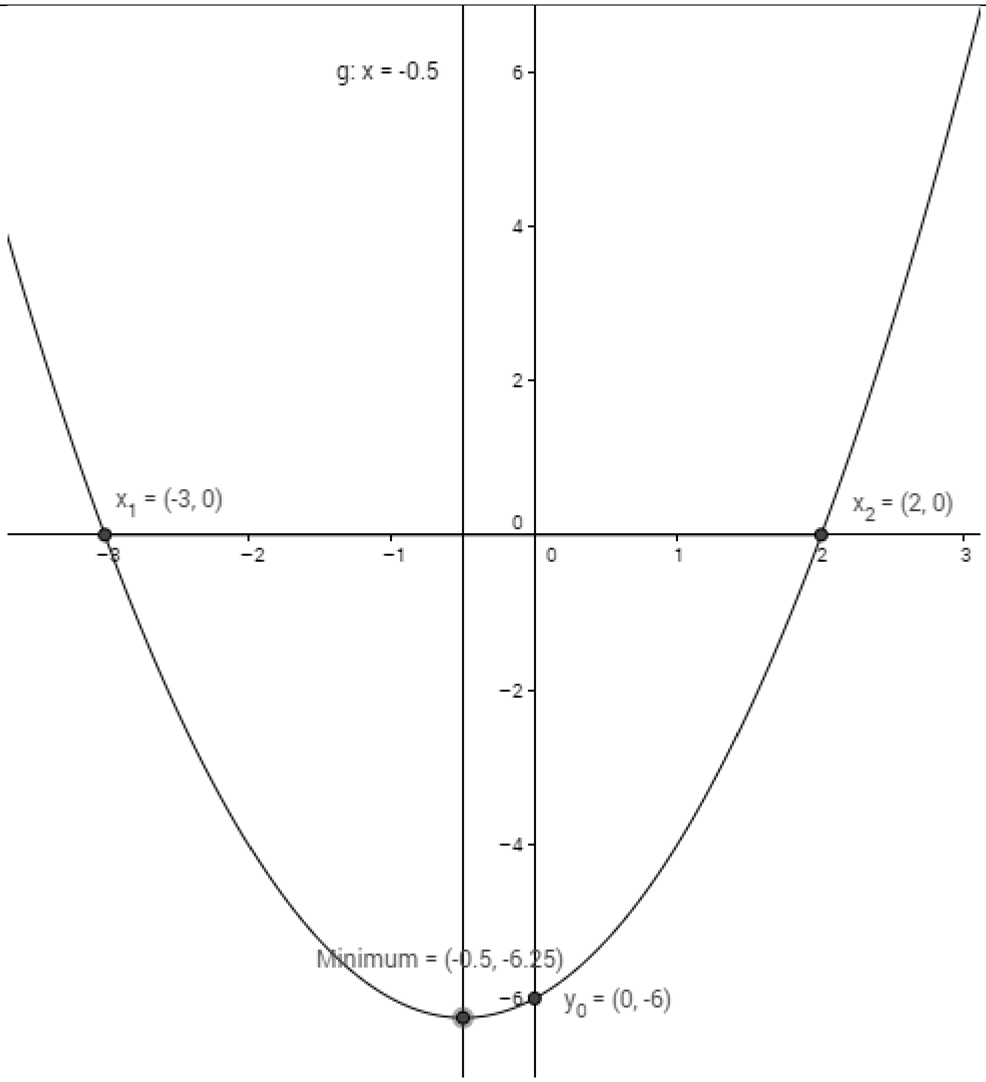
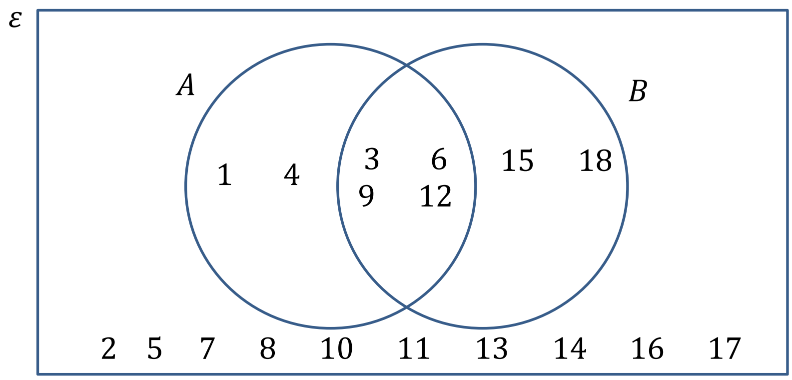
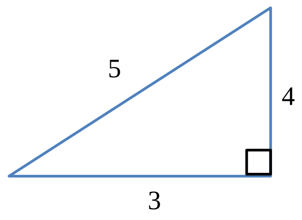
Solutions (Paper I)

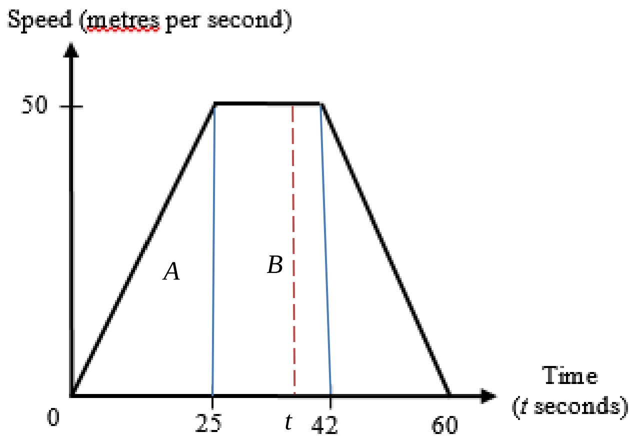
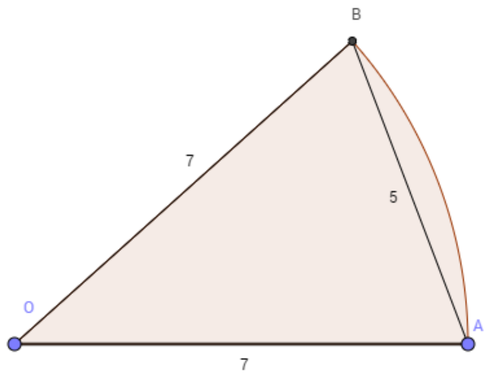
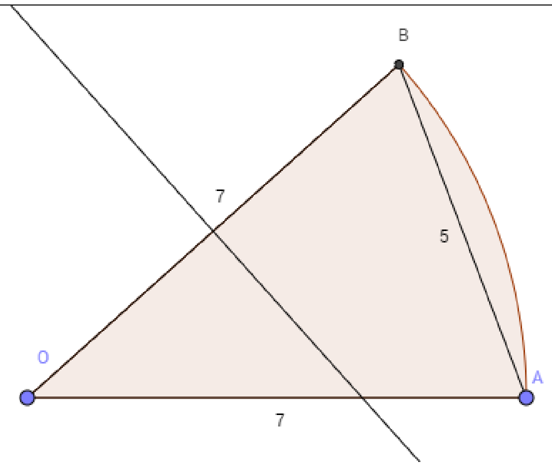
1	a	1.491
1	b	$\left(\frac{3x^2}{5yz}\right) \div \left(\frac{9x}{15y}\right)$ $= \left(\frac{3x^2}{5yz}\right) \times \left(\frac{15y}{9x}\right)$ $= \frac{x}{z}$
1	c	0.5 $0.5^2 = 0.25$ $\frac{5}{11} = 0.454545 \ldots \ldots$ $\frac{5}{9} = 0.555555 \ldots.$ $\therefore \quad 0.5^2 , \quad \frac{5}{11} , \quad 0.5 , \quad \frac{5}{9}$
2	a	$Difference = 7 - (-28) = 35°$
2	b	$1\ light\ year = 9.46 \times 10^{15}\ m = 9.46 \times 10^{12}\ km$ $Distance = 4.2 \times 9.46 \times 10^{12} = 39.732 \times 10^{12} = 3.9732 \times 10^{13}\ km$
3		Inconsistent scale on the vertical axis exaggerates the difference in passenger movement over the years.
4	a	$x^2 + 5x - 1$ $= \left(x + 5x + \left(\frac{5}{2}\right)^2\right) - \left(\frac{5}{2}\right)^2 - 1$ $= (x + 2.5)^2 - 7.25$
4	b	$x^2 + 5x - 1 = 0$ $(x + 2.5)^2 - 7.25 = 0$ $x + 2.5 = \pm\sqrt{7.25}$ $x = -2.5 \pm \sqrt{7.25}$ $x = 2.44 \ \ or \ \ -2.94$
5	a	$132 = 2^2 \times 3 \times 11$
5	b	$k = 2 \times 3^2 \times 11^2 = 2178$
6	a	$x^2 - 3 = q - 6x$ $q = (-2)^2 - 3 + 6(-2) = -11$
6	b	$x^2 - 3 = -11 - 6x$

		$x^2 + 6x + 8 = 0$ $x = -2\ (rejected)\ or - 4\ (ans)$
7	a	$\boldsymbol{N} = \boldsymbol{L} + \boldsymbol{M}^2$ $= \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} 6 & 0 \\ 3 & -4 \end{pmatrix}$ $= \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix} + \begin{pmatrix} 36 & 0 \\ 6 & 16 \end{pmatrix}$ $= \begin{pmatrix} 38 & 3 \\ -5 & 17 \end{pmatrix}$
7	b i	$\begin{pmatrix} 5 & 2 & 7 \\ 4 & 6 & 6 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$ $= \begin{pmatrix} 17 \\ 18 \end{pmatrix}$
7	b ii	The matrix represents the total no of points each team was awarded. Wochester got 17 points and Stomapool got 18 points.
8	a	

8	b	i	
8	b	ii	$x = -0.5$
9	a		$\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$ $A = \{1, \mathbf{3}, 4, \mathbf{6}, \mathbf{9}, \mathbf{12}\}$ $B = \{\mathbf{3}, \mathbf{6}, \mathbf{9}, \mathbf{12}, 15, 18\}$ 
9	b	i	2
9	b	ii	$\{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19\}$

10	$7-5=2\ units \rightarrow 12\ sweets$ $1\ unit \rightarrow 6\ sweets$ $Cindy, 11\ units \rightarrow 11\times 6=66\ sweets$
11 a	$\frac{1}{2}(x-16)<7x\leq 4x+14$ $\frac{1}{2}(x-16)<7x$ $x-16<14x$ $13x>-16$ $x>-\frac{16}{13}$ $or\ \ 7x\leq 4x+14$ $3x\leq 14$ $x\leq \frac{14}{3}$ $-\frac{16}{13}<x\leq\frac{14}{3}$
11 b	$biggest=4$ $smallest=-1$
12	$interior\ angle\ of\ hexagon=\frac{(6-2)\times 180}{6}=120°$ $angle\ HBC=360-90-120=150°$ $\frac{(n-2)\times 180}{n}=150$ $180n-360=150n$ $30n=360$ $n=12$
13	$Vol\ of\ solid\ hemisphere=\frac{1}{2}\left(\frac{4}{3}\right)\pi(7)^3=\frac{686}{3}\pi\ cm^3$ $Vol\ of\ hemisphere\ dug\ out=\frac{1}{2}\left(\frac{4}{3}\right)\pi(3.5)^3=\frac{343}{12}\pi\ cm^3$ $Vol\ of\ object=\frac{686}{3}\pi-\frac{343}{12}\pi=\frac{2401}{12}\pi\approx 629\ cm^3\ (3sf)$
14 a	$Total\ amount=20000\left(1+\frac{3.5}{100}\times\frac{1}{4}\right)^{\frac{9}{4}\times 4}\approx \$21631.27\ (2dp)$
14 b i	$Down\ payment=20\%\ of\ \$1250=\$250$
14 b ii	$Balance\ to\ be\ repaid=(1250-250)\times 103.5\%=\$1035$ $Mthly\ instalment=\frac{1035}{12}=\$86.25$

15	a	$By\ Sine\ Rule,$ $\frac{CD}{\sin x} = \frac{24}{\sin y}$ $CD = \frac{24 \times \frac{1}{3}}{\frac{4}{5}}$ $CD = 10\ cm$
15	b	 $Since \sin y = \frac{4}{5}, therefore \cos y = \frac{3}{5}$ $\cos \angle ADC = -\cos y$ $= -\frac{3}{5}$
16	a	$Area\ of\ equilateral\ triangle\ face\ of\ pyramid$ $= \frac{1}{2}(7)(7)\sin 60°$ $= 21.21762239 \ldots$ $\approx 21.2\ cm^2$
16	b	$Total\ surface\ area\ of\ pyramid = 4(21.21762239) + 7^2 = 133.8704896$ $\approx 134\ cm^2$
17	a	5, 14, 29, 50
17	b	1, 10, 25, 46 $The\ new\ sequence\ is\ 4\ less\ for\ every\ term.$ $\therefore the\ nth\ term\ for\ new\ sequence\ is = 3n^2 + 2 - 4 = 3n^2 - 2$
18	a	$acceleration\ when\ t_{13} = \frac{50}{25} = 2\ ms^{-2}$
18	b	$speed\ when\ t_{13} = 2 \times 13 = 26\ ms^{-1}$

18 c	 $Distance\ travelled\ in\ first\ 25\ s = \frac{1}{2}(25)(50) = 0.625\ km$ $Distance\ travelled\ in\ part\ B = (42 - 25)(50) = 0.850\ km$ $Total\ distance\ A + B = 1.475\ km$ $Therefore\ time\ taken\ to\ travel\ the\ first\ 1.2\ km\ must\ be\ between\ 25\ and\ 42\ s.$ $Area\ of\ trapezium = 1.2\ km$ $\frac{1}{2}(t + t - 25)(50) = 1200\ m$ $2t - 25 = \frac{1200}{25}$ $t = 36.5\ s$
19 a	
19 b i	

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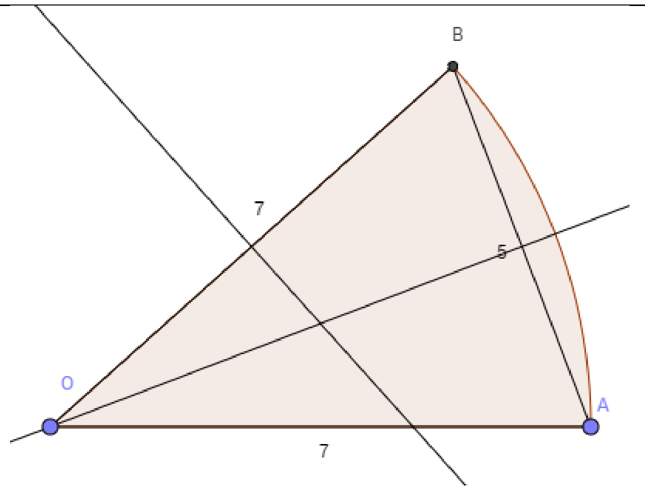
<tr><td>19 b ii</td><td>
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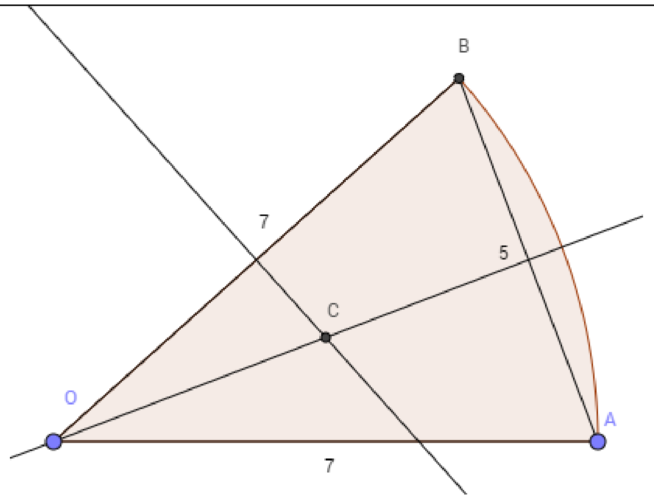
<tr><td>19 b iii</td><td>
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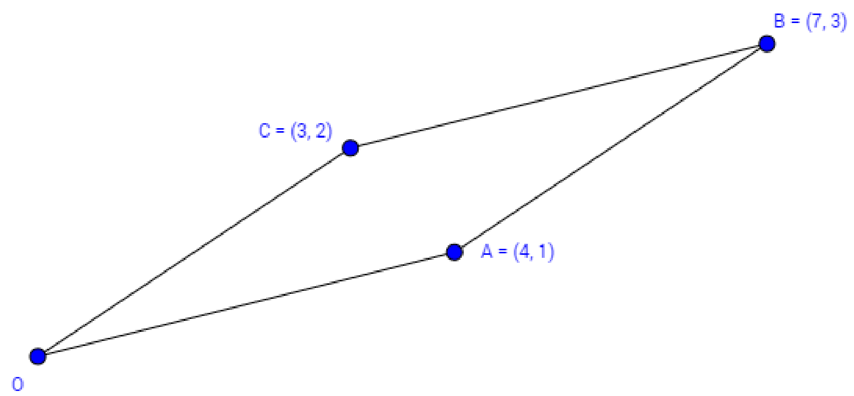
<tr><td>20 a</td><td>

$OC \text{ is } \parallel \text{ to } AB, \text{therefore } \overrightarrow{OC} = \overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$</td></tr>
<tr><td>20 b i</td><td>$\overrightarrow{AQ} = k\overrightarrow{CA}$

$\overrightarrow{OQ} - \overrightarrow{OA} = k\overrightarrow{CA}$

$\overrightarrow{OQ} = \overrightarrow{OA} + k\overrightarrow{CA}$

$\overrightarrow{OQ} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} + k\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 4+k \\ 1-k \end{pmatrix}$ *Shown*</td></tr>
</table>

20	b	ii	$Since\ Q\ lies\ on\ x-axis,$ $\therefore 1-k=0$ $k=1$
20	b	iii	$\overrightarrow{OQ}=\begin{pmatrix}4+1\\1-1\end{pmatrix}=\begin{pmatrix}5\\0\end{pmatrix}$ $Coordinates\ of\ Q\ is\ (5,0).$
21	a	i	$No\ of\ students\ who\ scored\ less\ than\ 50\ marks=60$ $\frac{60}{160}\times 100\%=37.5\%$
21	a	ii	$No\ of\ students\ who\ scored\ less\ than\ 74\ marks=132$ $No\ of\ students\ who\ scored\ distinctions=160-132=28$
21	a	iii	$Q_1=43$ $Q_2=56$ $Q_3=68$ $IQR=68-43=25\ marks$
21	b		$Maths\ was\ easier\ as\ the\ median\ mark\ for\ Maths\ is\ higher.$
22	a		$Equation\ of\ line\ AB: y-3=\left(\frac{-2-3}{-3-5}\right)(x-5)$ $y=\frac{5}{8}x-\frac{1}{8}$
22	b		$\frac{3}{2}=\frac{5}{8}k-\frac{1}{8}$ $k=\left(\frac{3}{2}+\frac{1}{8}\right)\left(\frac{8}{5}\right)=\frac{13}{5}$
22	c		$Length\ of\ AB=\sqrt{(-3-5)^2+(-2-3)^2}\approx 9.43\ units$
22	d		$gradient\ of\ 3x+y=6\quad is-3$ $New\ line\ equation:$ $y=mx+c$ $3=-3(5)+c$ $c=11$ $\therefore equation\ is\ y=-3x+11$

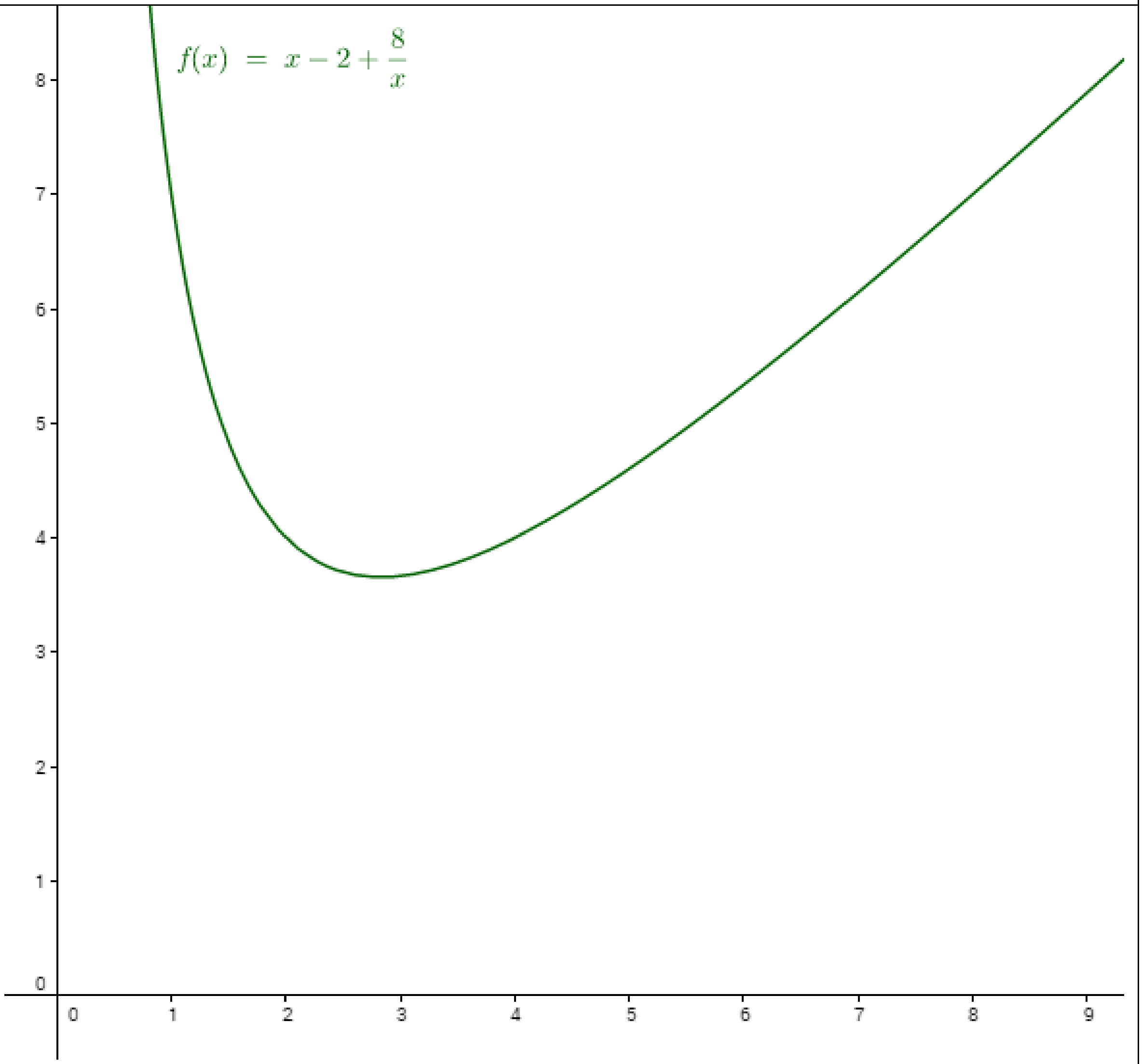
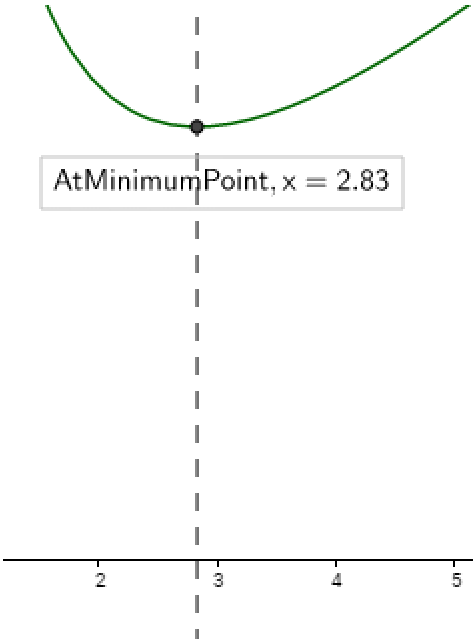
Solutions (Paper II)

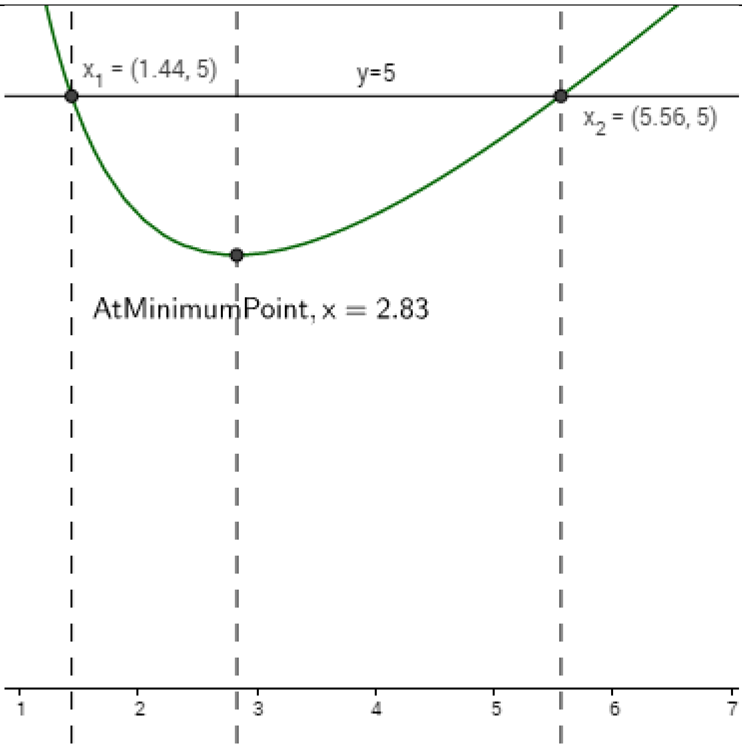
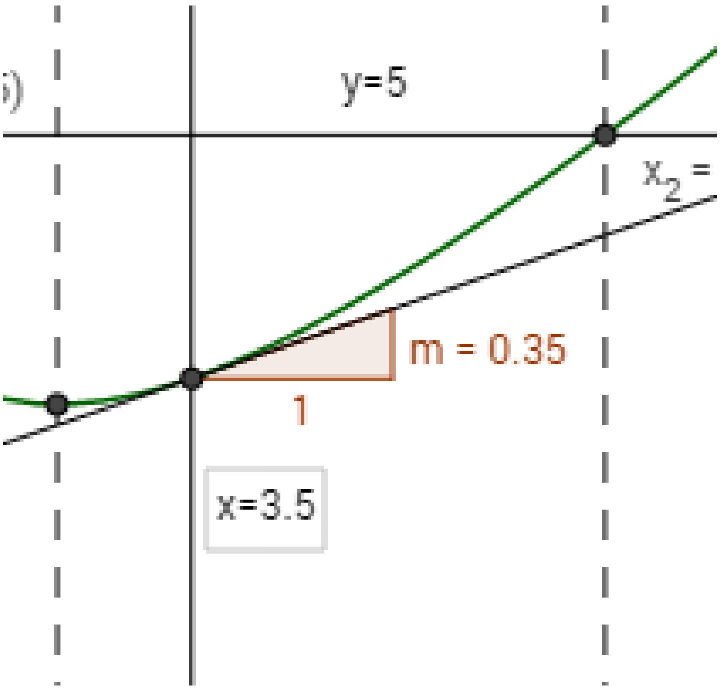
1	a	$\frac{2}{9a^2-16}-\frac{3}{4-3a}$ $=\frac{2}{(3a+4)(3a-4)}+\frac{3}{3a-4}$ $=\frac{2}{(3a+4)(3a-4)}+\frac{3(3a+4)}{(3a-4)(3a+4)}$ $=\frac{9a+14}{(3a+4)(3a-4)}$
1	b	$y=\frac{k}{x^2}$ $y_0=\frac{k}{x_0^2}\ \ for\ when\ x=x_0$ $k=y_0x_0^2$ $y=\frac{y_0x_0^2}{x^2}$ $When\ x=2x_0,$ $y=\frac{y_0x_0^2}{(2x_0)^2}$ $y=\frac{y_0x_0^2}{4x_0^2}=\frac{1}{4}\ y_0$ $Hence, percentage\ decrease\ is\ \frac{1-0.25}{1}\times 100\%=75\%$
1	c	$x=\frac{3y+2}{x-1}$ $x^2-x-2=3y$ $\left(x-\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-2=3y$ $\left(x-\frac{1}{2}\right)^2=3y-2\frac{1}{4}$ $x=\frac{1}{2}\pm\sqrt{3y-2\frac{1}{4}}$
2	a	$Income\ tax\ reliefs=1000+2000+4(4000)+2(7000)+22040=\$55040$
2	b	$Chargeable\ income=total\ income-reliefs$ $=7600(14.5)-55040$ $=\$55160$
2	c	$Based\ on\ 3rd\ row\ in\ the\ table,$ $Income\ tax=900+8.5\%\times(55040-40000)=\$2178.40$

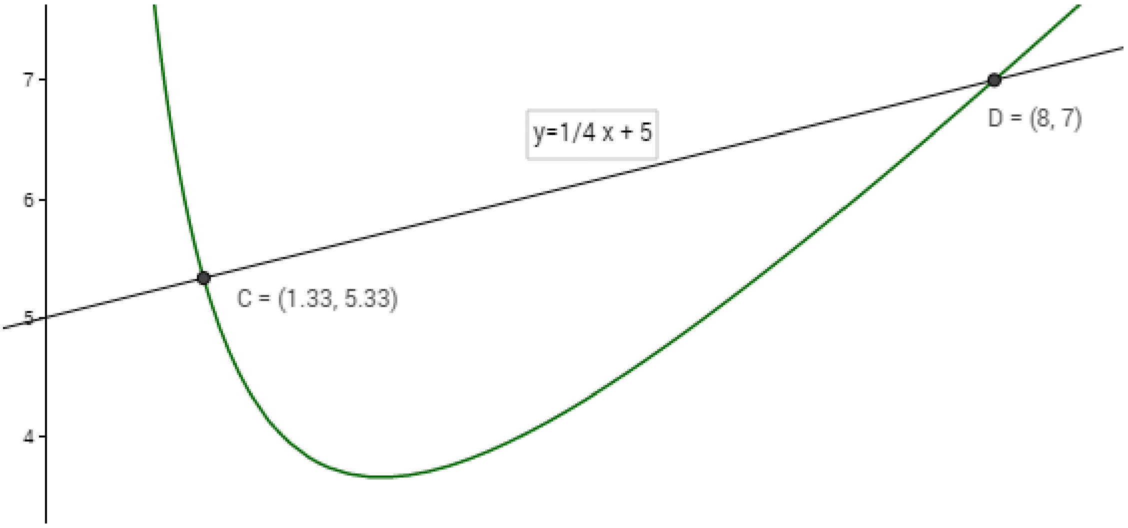
2 d	$Based\ on\ 3rd\ row,$ $first\ part\ of\ income\ tax = \$900$ $\therefore first\ part\ of\ chargeable\ income = \$40000$ $second\ part\ of\ income\ tax = 3280 - 900 = \$2380$ $\therefore second\ part\ of\ chargeable\ income \times 8.5\% = 2380$ $second\ part\ of\ chargeable\ income = \frac{2380}{8.5\%} = \$28000$ $Total\ chargeable\ income = \$40000 + 28000 = \$68000$
3 a	$(x + 1.5)$ km/h
3 b i	$Ivy's\ time = \frac{5}{x}\ hours$
3 b ii	$Pauline's\ time = \frac{3}{x} + \frac{2}{x + 1.5}\ hours$
3 c	$Ivy's\ time - Pauline's\ time = 20\ minutes$ $\frac{5}{x} - \left(\frac{3}{x} + \frac{2}{x + 1.5}\right) = \frac{20}{60}$ $\frac{2}{x} - \frac{2}{x + 1.5} = \frac{1}{3}$ $\frac{2(x + 1.5) - 2x}{x(x + 1.5)} = \frac{1}{3}$ $3 \times 3 = x^2 + 1.5x$ $x^2 + \frac{3}{2}x - 9 = 0$ $2x^2 + 3x - 18 = 0\ \ Hence\ shown$
3 d	$2x^2 + 3x - 18 = 0$ $x = 2.34233 \ldots \approx 2.342, \qquad or\ x = -3.842\ (rejected)$
3 e i	$Pauline's\ time = \frac{3}{2.34233} + \frac{2}{2.34233 + 1.5} = 1.80129\ hours \approx 1\ hr\ 48\ min$
3 e ii	$Pauline's\ average\ speed = total\ distance \div total\ time = \frac{5}{1.80129} \approx 2.78$ km/h
4 a	$area\ of\ shaded\ sector\ AOB = 120\ cm^2$ $\frac{1}{2}(7)^2(reflex\ angle\ AOB) = 120$ $reflex\ angle\ AOB = \frac{120 \times 2}{49} = \frac{240}{49} \approx 4.89\ radians$
4 b	$Major\ Arc\ Length\ AOB = (7)\left(\frac{240}{49}\right) = 34\frac{2}{7}$ $perimeter\ of\ AOB = 34\frac{2}{7} + 7 + 7 = 48\frac{2}{7}\ cm$
4 c	$Area\ of\ minor\ sector\ AOB = \frac{1}{2}(7)^2\left(2\pi - \frac{240}{49}\right) = 33.93804\ cm^2$ $Area\ of\ triangle\ AOB = \frac{1}{2}(7)^2 sin\left(2\pi - \frac{240}{49}\right) = 24.07936\ cm^2$ $Area\ of\ segment\ AMBX = 33.93804 - 24.07936 \approx 9.86\ cm^2$

4	d		$Angle\ AOB = 2\pi - \frac{240}{49} = 1.385$ $In\ triangle\ MOB,$ $\cos\left(\frac{1}{2}A\hat{O}B\right) = \frac{OM}{7}$ $OM = 7\cos\frac{1}{2}(1.385) = 5.387\ cm$ $MX = 7 - 5.387 \approx 1.61\ cm\ (3sf)$
5	a		$Area\ of\ big\ capsule\ shape = 130 \times (48 + 8 + 8) + (\pi)(24 + 8)^2 = 8320 + 1024\pi\ m^2$ $Area\ of\ small\ capsule\ shape = 130 \times (48) + (\pi)(24)^2 = 6240 + 576\pi\ m^2$ $Area\ of\ running\ track = 2080 + 448\pi\ m^2$
5	b		$each\ lane\ is\ 1\ m\ wide. mid\ of\ 3rd\ lane\ means\ 2.5\ m\ from\ edge\ of\ field.$ $each\ lap\ distance\ for\ woman = 130(2) + 2(\pi)(24 + 2.5) = 260 + 53\pi\ m = \frac{260 + 53\pi}{1000}\ km$ $Total\ distance\ ran = 6\left(\frac{260 + 53\pi}{1000}\right) \approx 2.56\ km\ (3sf)$
5	c		$each\ lap\ distance\ for\ man = 260 + 53\pi\ m$ $Total\ distance\ ran = speed \times time = 12.2 \times (2(60) + 35\ s) = 1891\ m$ $No. of\ laps\ ran = 1891 \div (260 + 53\pi) \approx 4.4\ laps\ (1\ d.p.)$
6	a		$x - 5\ years$
6	b		$y - 5\ years$
6	c		$x - 5 = 4(y - 5)$
6	d		$x + y = 35$
6	e		$x + y = 35$ $y = 35 - x \ldots\ldots (1)$ $x - 5 = 4(y - 5) \ldots\ldots (2)$ $Sub\ (1)\ into\ (2):$ $x - 5 = 4(35 - x - 5)$ $5x = 125$ $x = 25$ $y = 35 - 25 = 10$ $Hence\ Shawn\ is\ 25\ years\ and\ Jasper\ is\ 10\ years.$
7	a	i	$angle\ ADC = \sin^1\left(\frac{54\sin 68°}{74}\right) = 42.57803 \ldots \approx 42.6°$
7	a	ii	$angle\ ABC = \cos^{-1}\left(\frac{63^2 + 92^2 - 54^2}{2(63)(92)}\right) = 34.81525 \ldots\ldots \approx 34.8°$
7	a	iii	The bearing of B from A = 360 – 90 – 34.8 = 235.2°
7	a	iii	$Area\ of\ triangle\ ABC = \frac{1}{2}(63)(92)\sin 34.81525°$ = 1654.56 $Area\ of\ triangle\ ADC = \frac{1}{2}(54)(74)\sin(180 - 42.578 - 68)° =\ 1870.52$ Area of quad ABCD = 3530 m^2

7	b	$\cos 34.18525° \ = \frac{b}{63}$ $Distance\ walked\ from\ B, b = 63\ \ \cos 34.18525° = 52.115 \approx 52.1m$
7	c	$\tan 42° = \frac{AF}{63}$ $AF = 63 \tan 42° = 56.72545 \ldots \approx 56.7\ m$

8 a	$k = 7 - 2 + \frac{8}{7} = 6.14286 \ldots \approx 6.1\ (1\ d.p.)$
8 b	
11 c i	 Accept 2.8, 2.85, 2.9

8 c ii	 Accept 1.4, 1.45, 1.5 and 5, 5.5, 5.6
8 d	 *gradient when* $x = 3.5$ *is* $0.35\ \pm 0.1$.

8	e	$4\left(x-2+\frac{8}{x}\right)=x+20$ $x-2+\frac{8}{x}=\frac{1}{4}x+5$ $to\ plot\ graph\ of\ y=\frac{1}{4}x+5$  Accept 1.25, 1.3, 1.35, 1.4 and 7.9, 7.95, 8, 8.05, 8.1
9	a	$angle\ QRS=90°\ (angle\ in\ semicircle)$ $angle\ SQR=180-90-24=66°\ (angle\ sum\ of\ triangle)$
9	b	$angle\ ORS=angle\ OSR=24°\ (angles\ of\ isos.triangle)$ $angle\ ROS=180-24-24=132°\ (angle\ sum\ of\ triangle)$
9	c	$angle\ SPR=180-angle\ SQR\ (angles\ in\ opp.segments)$ $=180-66=114°$
9	d	$angle\ ARS=180-angle\ QRS=90°\ (angles\ on\ a\ str.line)$ $angle\ ASR=180-90-38=52°\ (angle\ sum\ of\ triangle)$ $angle\ PRS=180-angle\ SPR-angle\ ASR=180-114-52=14°$ $\therefore angle\ PRQ=angle\ PRS+angle\ QRS=14+90=104°$
9	e	$angle\ PRO=angle\ PRS+angle\ ORS=14+24=38°$
10	a	$w=\frac{5}{20}=\frac{1}{4}$ $x=\frac{8}{19}$ $y=\frac{6}{19}$ $z=\frac{4}{19}$
10	b i	$P(both\ balls\ same\ colour)$ $=P(RR)\ or\ P(BB)\ or\ P(YY)$ $=\frac{9}{20}\times\frac{8}{19}+\frac{6}{20}\times\frac{5}{19}+\frac{1}{4}\times\frac{4}{19}=\frac{61}{190}$
10	b ii	$P(both\ balls\ different\ colour)$ $=1-P(both\ balls\ same\ colour)$

			$= 1 - \frac{61}{190} = \frac{129}{190}$
10	b	iii	$P(one\ ball\ is\ red\ and\ another\ is\ blue)$ $= P(RB)\ or\ P(BR)$ $= \frac{9}{20} \times \frac{6}{19} + \frac{6}{20} \times \frac{9}{19} = \frac{27}{95}$
10	c		$P(none\ of\ 3\ balls\ are\ yellow)$ $= 1 - P(Y) - P(RY) - P(BY) - P(RRY) - P(RBY) - P(BRY) - P(BBY)$ $= 1 - \left(\frac{1}{4}\right) - \left(\frac{9}{20} \times \frac{5}{19}\right) - \left(\frac{6}{20} \times \frac{5}{19}\right) - \left(\frac{9}{20} \times \frac{8}{19} \times \frac{5}{18}\right) - \left(\frac{9}{20} \times \frac{6}{19} \times \frac{5}{18}\right) - \left(\frac{6}{20} \times \frac{9}{19} \times \frac{5}{18}\right)$ $- \left(\frac{6}{20} \times \frac{5}{19} \times \frac{5}{18}\right)$ $= 1 - \frac{1}{4} - \frac{9}{76} - \frac{3}{38} - \frac{1}{19} - \frac{3}{76} - \frac{3}{76} - \frac{5}{228} = \frac{91}{228}$
11	a		Max possible mass of empty tank = $\frac{1}{3}(96000)$ = 32000 kg
11	b		Volume of tank = $\frac{1}{3}\pi\left(\frac{d}{2}\right)^2(0.5d) + \pi\left(\frac{d}{2}\right)^2(2.5d)$ $62.3 \times 1000^3 \div 90\% = \frac{2}{3}\pi d^3$ $d^3 = (62.3 \times 1000^3 \div 90\%) \div \frac{2}{3}\pi$ $d = \sqrt[3]{33\ 051\ 176\ 515}$ = 3 210 (3 s.f.)
11	c		Volume of interior of tank = 62.3 ÷ 90% = 69.2 m^3 For 96 000 kg, Max Vol of cement = 96000 ÷ 1410 = 68.085 63.2 < 68.085 < 69.2 Min Vol of cement = 96000 ÷ 1506 = 63.745 63.2 < 63.745 < 69.2 It is possible to fill the tank with 96 000 kg of cement since the volme is less than the volume of the interior of the tank. However, it is not advisible as the volume of 96 000 kg of cement exceeds the capacity of the tank.