Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

This paper consists of 16 printed pages.

For Examiner's Use

Answer **all** questions

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1 **(a)** Calculate $\dfrac{9.645^3}{\sqrt[4]{6.55}+\sqrt{49.5}}$, giving your answer correct to 4 significant figures.

Answer (*a*) **103.9** [1]

(b) Simplify $\left(\dfrac{1}{8x^{-6}y}\right)^{\frac{2}{3}} \times \left(\dfrac{y}{x^{-2}}\right)^{0} \div \left(\dfrac{1}{y}\right)^{-\frac{1}{3}}$.

$$\left(\frac{1}{8x^{-6}y}\right)^{\frac{2}{3}} \times \left(\frac{y}{x^{-2}}\right)^{0} \div \left(\frac{1}{y}\right)^{-\frac{1}{3}} = \frac{x^4}{4y^{\frac{2}{3}}} \times 1 \times y^{-\frac{1}{3}}$$

$$= \frac{x^4}{4y}$$

Answer (*b*) $\dfrac{x^4}{4y}$ [2]

(c) The number 2003.09 can be written as

$$2\times10^3 + 3\times10^x + 9\times10^y.$$

Given that x and y are integers, find the values of x and y.

Answer (*c*) $x=0$, $y = -2$ [2]

2 **(a)** Calculate the difference of 4.6×10^6 and 3.7×10^5. Give your answer in standard form, to 2 significant figures.

$$4.6\times10^6 - 3.7\times10^5 = (46-3.7)\times10^5$$

$$= 4.2\times10^6$$

Answer (*a*) 4.2×10^6 [2]

(b) The area of Singapore is about 710 km^2. Express the area in square metres, giving your answer in standard form.

Answer (*b*) 7.10×10^8 **m^2** [1]

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3 Given that $p = 9.25 \times 10^{-99}$, $q = 4.76 \times 10^{-96}$, express $\frac{q}{p}$ in standard form.

$$\frac{q}{p} = (4.76 \times 10^{-96}) \div (9.25 \times 10^{-99})$$
$$= (4.76 \div 9.25) \times 10^{(-96)-(-99)}$$
$$= 0.51459 \times 10^{3}$$
$$= 5.1459 \times 10^{-1} \times 10^{3}$$
$$= 5.15 \times 10^{2}$$

Answer 5.15×10^{2} [1]

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4 **(a)** Express $-m^2 + 6m - 13$ in the form $a(m-b)^2 + c$.

$$-m^2 + 6m - 13 = -(m^2 - 6m) - 13$$
$$= -(m^2 - 6m + 9 - 9) - 13$$
$$= -(m-3)^2 - 4$$

Answer (*a*) $-(m-3)^2 - 4$ [2]

(b) **Hence**, solve the equation $-m^2 + 6m - 13 = 0$, giving your answers correct to two decimal places .

$$-m^2 + 6m - 13 = 0$$
$$-(m-3)^2 - 4 = 0$$
$$m - 3 = \pm\sqrt{\frac{4}{2}}$$
$$m = 1,5$$

Answer (*b*) $m = 1,\ 5$ [2]

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5 Expressed as the product of prime factors,

$$132 = 2^2 \times 3 \times 11 \quad \text{and} \quad 42 = 2 \times 3 \times 7$$

Use the results to find

(a) the smallest integer, *k*, such that 132*k* is a perfect cube.

Answer (*a*) **2178** [1]

(b) the smallest positive integer, *n*, such that 42*n* is a multiple of 132.

Answer (*b*) **n=22** [1]

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6 One solution of $2x^2 - 7 = a - 5x$ is $x = -4$.

Find

(a) the value of *a*,

$2(-4)^2 - 7 = a - 5(-4)$

$a = 5$

Answer a= **5** [1]

(b) the other solution of the equation.

$2x^2 - 7 = 5 - 5x$

$2x^2 + 5x - 12 = 0$

$(x + 4)(2x - 3) = 0$

$x = -4,$

$x = 1\frac{1}{2}$

Answer : $x = 1\frac{1}{2}$ [2]

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7 **(a)** If $\mathbf{A} = \begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & 5 \\ 0 & -6 \end{pmatrix}$, evaluate $\mathbf{C} = \mathbf{A}^2 + \mathbf{B}$.

$$\mathbf{C} = \mathbf{A}^2 + \mathbf{B}$$
$$= \begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 4 & 5 \\ 0 & -6 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & -12 \\ 4 & 1 \end{pmatrix} + \begin{pmatrix} 4 & 5 \\ 0 & -6 \end{pmatrix}$$
$$= \begin{pmatrix} 5 & -7 \\ 4 & -5 \end{pmatrix}$$

Answer $\begin{pmatrix} 5 & -7 \\ 4 & -5 \end{pmatrix}$ [2]

(b) A large cinema has a total of 20 theatres consisting of 2 theatres of type *A*, 3 of type *B*, 6 of type *C* and 9 of type *D*. The theatre has 3 classes of seat known as Economy, Executive and VIP. The table below shows the number of these seats in each of the 4 types of theatres.

	Economy	Executive	VIP
A	0	30	20
B	50	30	10
C	70	20	5
D	100	10	0

(i) Write down two matrices whose product shows the total number of seats in each class.

$$\begin{pmatrix} 0 & 50 & 70 & 100 \\ 30 & 30 & 20 & 10 \\ 20 & 10 & 5 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 6 \\ 9 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 2 & 3 & 6 & 9 \end{pmatrix}\begin{pmatrix} 0 & 30 & 20 \\ 50 & 30 & 10 \\ 70 & 20 & 5 \\ 100 & 10 & 0 \end{pmatrix}$$

Answer (bi) **……** [2]

(ii) Evaluate this product of matrices.

$$\begin{pmatrix} 1470 \\ 360 \\ 100 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 1470 & 360 & 100 \end{pmatrix}$$

Answer (bii) **……** [1]

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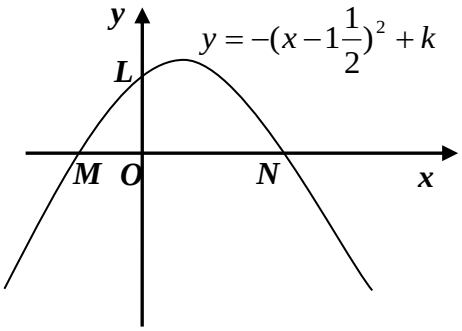
8 **(a)** The diagram shows the graph of $y = -(x - 1\frac{1}{2})^2 + k$.



(i) Write down the line of symmetry

Answer (*ai*) $x = 1\frac{1}{2}$ [1]

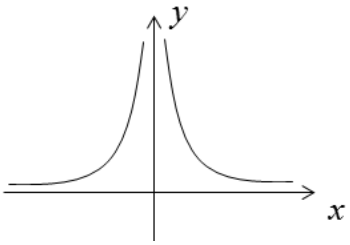
(ii) Given that the graph cuts the y-axis at L (4, 0), cuts the x-axis at M (m, 0) and N (n , 0). Find the value of m and n.

$$y = -(x - 1\frac{1}{2})^2 + \frac{25}{4}$$

when $y = 0$, $x = -1, 4$

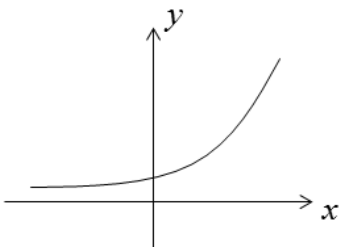
Answer (*aii*) **m= -1, n=4** [2]

(b) The sketch represents the graph of $y = x^n$. Write down a possible value of n.



Answer (*b*) **Any negative even integer** [1]

(c) Write down a possible equation for this graph.



Answer (*c*) $y = a^x$ (any a value for $a > 1$)

For Examiner's Use

9 Of the 38 students in a class, 23 have *Line* account and 15 have *WeChat* account. It is given that

$\varepsilon = \{\text{students in the class}\}$,

$A = \{\text{students who have Line account}\}$

and $B = \{\text{students who have WeChat account}\}$.

Let $n(A \cap B) = x$.

(a) Express $n(A \cup B)$ in terms of x.

$$n(A \cup B) = 23 - x + x + 15 - x$$
$$= 38 - x$$

Answer (*a*) **38 - *x*** [2]

(b) Find the smallest and largest possible values of x.

Answer (*b*) Smallest possible value of x is **0** [1]

Largest possible value of x is **15** [1]

10 Given that $a : b = 6 : 11$ and $a : c = 5 : 7$, find $a : b : c$.

$a : b : c$

$5 \times 6 : 11 \times 5$

$6 \times 5 : 7 \times 6$

$a : b : c = 30 : 55 : 42$

Answer $a : b : c = 30 : 55 : 42$ [2]

For Examiner's Use

11 **(a)** Solve $-4 \le -7x + 10 < 31$.

$-4 \le -7x + 10 < 31$

$-14 \le -7x < 21$

$-3 < x \le 2$

Answer (*a*) $-3 < x \le 2$ [2]

(b) Write down the greatest and least integers which satisfy $-4 \le -7x + 10 < 31$.

Answer (*b*) **2** and **-2**

12 An interior angle of a regular *n*-sided polygon is larger than its exterior angle by 50%. Find *n*.

$\frac{(n-2) \times 180°}{n} = 150\% \times \frac{360°}{n}$

$n = 5$

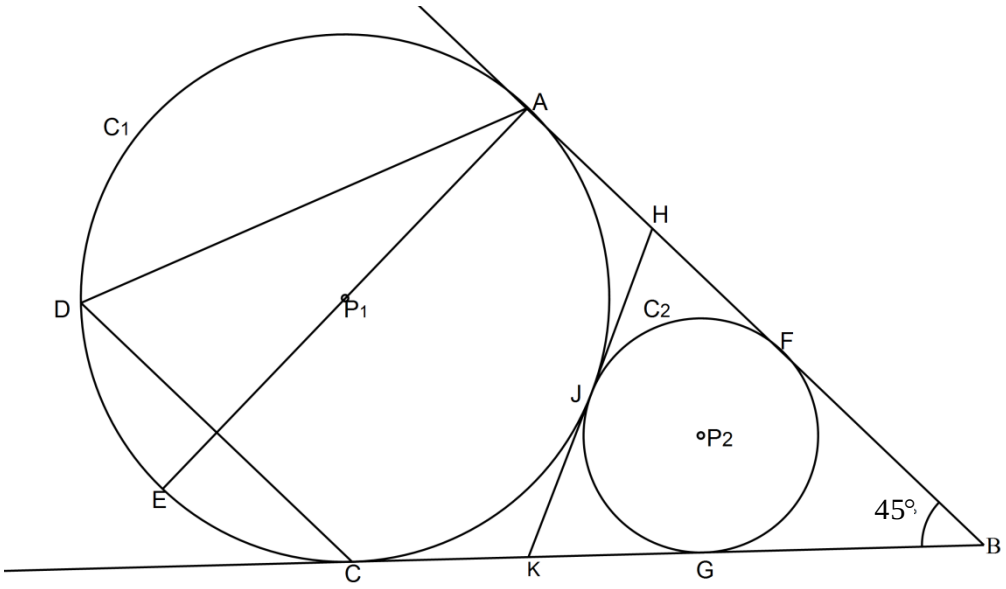
Alternative solution

Let x = ext angle
1.5 x = int angle
$x + 1.5x = 180$
$x = 72^o$
no of sides = 360/72 = 5

Answer n = **5** [2]

For Examiner's Use

13 The diagram shows two circles, C_1 and C_2, with centres P_1 and P_2 respectively. The two circles meet at *J*. *HJK* is a tangent to both circles. *AE* is a diameter of the bigger circle.



(a) Given that the tangents of circle C_1 at *A* and *C* meet at *B*. $\angle ABC = 45°$

Showing your working, calculate

(i) angle *ADC*

$\angle P_1CB = \angle P_1AB = 90°$ (tangent $\perp$ radius)

$\angle AP_1C = 360° - 90° \times 2 - 45° = 135°$

$\angle ADC = \frac{1}{2}\angle AP_1C = 67.5°$ ($\angle$ at center $= 2\angle s$ at circumference)

Answer (*a*)(*i*) 67.5° [2]

(ii) angle *AEC*

$\angle AEC = \angle ADC = 67.5°$ ($\angle$s in same segment)

Answer (*a*)(*ii*) 67.5° [1]

(b) Circle C_2 passes through *J, F* and *G*. *AFB, CGB* and *HJK* are tangents to the circle C_2.

(i) Given that *HB = 7 cm*, find the length of *BK*.

AB = BC , BF =BG (tangent from ext point)

$AB - BF = BC - BG$

$\therefore$ AF = CG

AH = HJ = HF , CK = KJ = KG (tangent from ext point)

$KG = \frac{1}{2}CG = \frac{1}{2}AF = HF$

BK = BG+GK = HF+BF =BF=7 cm

Answer (*b*)(*i*) **7 cm** [2]

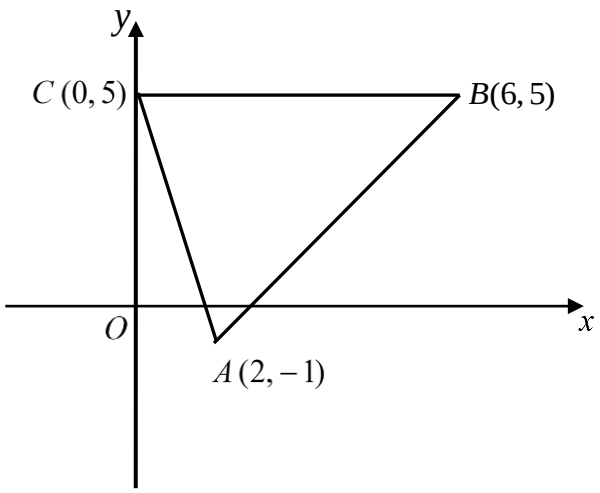
(ii) Hence, or otherwise, write down angle *BHK* .

$\angle BHK = \frac{180° - 45°}{2} = 67.5°$ (base $\angle$s of isos Δs)

Answer (*b*)(*ii*) 67.5° [1]

For Examiner's Use

14 The vertices of triangle ABC are $A(2,-1)$, $B(6,5)$ and $C(0,5)$.



(a) Find the area of triangle ABC.

$$\text{area} = \frac{1}{2} \times 6 \times 6 = 18 unit^2$$

Answer (*a*) **18 units^2** [1]

(b) CX is a line perpendicular to AB and X is a point on AB. Using your answer from part (a), find the length of CX.

$$\text{length of AB} = \sqrt{(6-2)^2 + (5-(-1))^2}$$

$$= \sqrt{52}$$

$$\text{length of CX} = \frac{18}{\frac{1}{2} \times \sqrt{52}}$$

$$= 4.99$$

Answer(b) 4.99 units [3]

(c) Find $\angle ACB$.

$\text{BC} = 6$,

$\text{AC} = \sqrt{2^2 + 6^2} = \sqrt{40}$ or $2\sqrt{10}$, $\text{AB} = \sqrt{4^2 + 6^2} = \sqrt{52}$ or $2\sqrt{13}$

$$\cos \angle \text{ACB} = \frac{6^2 + (\sqrt{40})^2 - (\sqrt{52})^2}{2 \times 6 \times \sqrt{40}}$$

$\angle \text{ACB} = 71.6°$

Answer (*c*) $\angle ACB = 71.6°$ [3]

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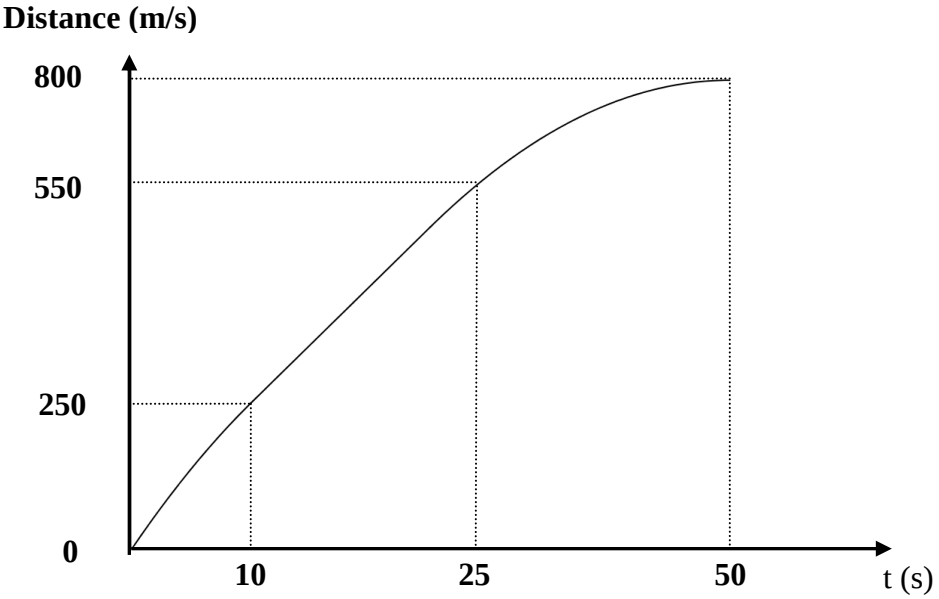
15 The diagram shows the distance-time graph for the first 50 seconds of a car's journey. The car came to a rest after 50 seconds.



(a) Find the speed of the car when $t = 15$

$$Speed = \frac{550 - 250}{25 - 10} = 20 \text{ m/s}$$

Answer (*a*) 20 m/s [1]

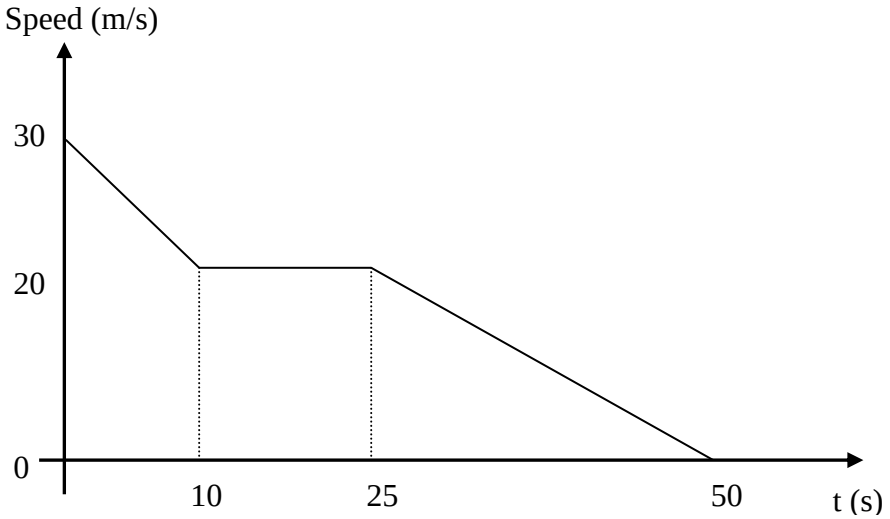
(b) Given that the car moved with a constant deceleration for the last 25 seconds. Find its deceleration for the last 25 seconds.

Answer (*b*) *Deceleration for the last 25s is* ***0.8*** m/s^2 [1]

(c) Given that the car also moved with a constant deceleration for the first 10 seconds, and the initial speed is 30 m/s.

On the axes given, sketch the speed-time graph for the car's journey.

Answer (c)



[3]

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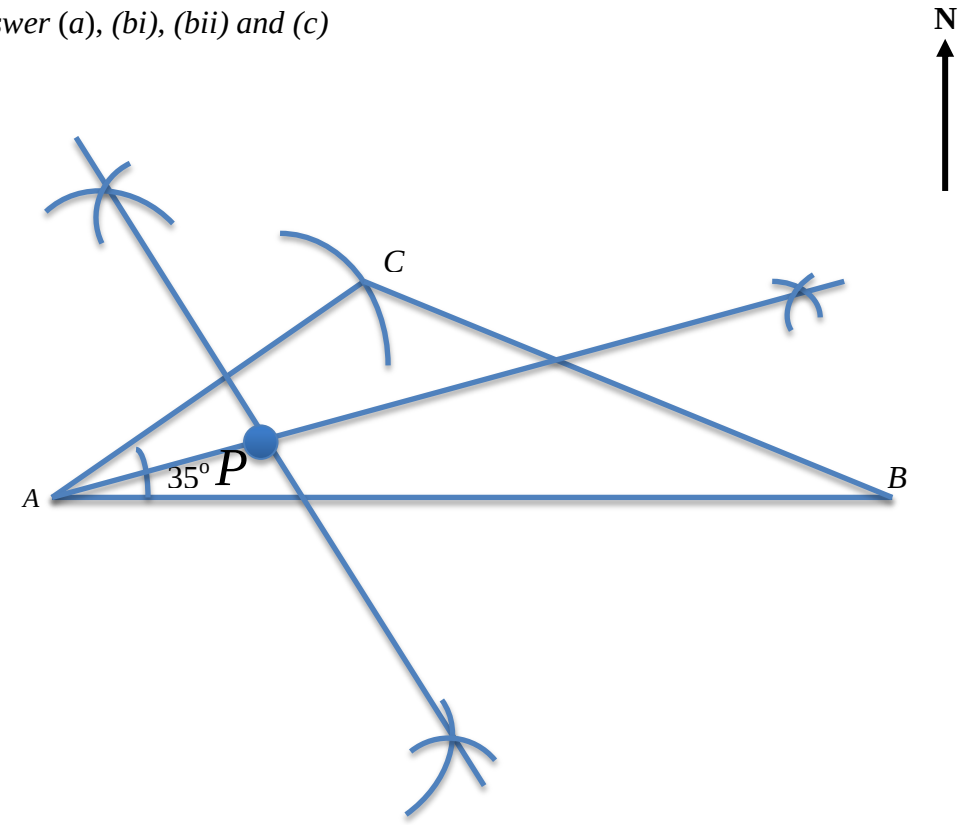
16 A scale map drawing shows the positions of the towns *A*, *B* and *C*. Given that *AB*= 11 cm, *AC* = 5 cm , *B* is due East of *A* and *C* is on a bearing of 055° from *A*.

Answer (*a*), *(bi), (bii) and (c)*



(a) Construct the triangular region *ABC*. [2]

(b) In the ΔABC **,** construct

(i) the perpendicular bisector of line *AC*, [1]

(ii) the angle bisector of $\angle CAB$, [1]

(c) Mark and label the point *P* inside ΔABC that is equidistant from the lines *AC* and *AB* and is also equidistant from town *A* and town *C*. [1]

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(d) Find the bearing of *A* from *C*.

Answer (d) 235^{o} [1]

The map is drawn to a scale of 1:400 000.

(e) **(i)** Calculate the actual distance between town *A* and town *B*.

$1\,\text{cm} : 4\,\text{km}$

$11\text{cm} : 44\,\text{km}$

Answer (ei) **44 km** [2]

(ii) Calculate the actual area of the triangular region *ABC*.

$1\,\text{cm}^2 : 16\,\text{km}^2$

$15.773\,\text{cm}^2 : 252\,\text{km}^2$ (area of the triangle can be found out either by calculation or by measurement)

Answer (eii) **252** km^2 [2]

For Examiner's Use

18 The terms T_1, T_2, T_3, T_4 of a sequence are given as follows:

$$T_1 = 1 = 1$$
$$T_2 = 9 = 1 + 8$$
$$T_3 = 25 = 1 + 8 + 16$$
$$T_4 = 49 = 1 + 8 + 16 + 24$$

(a) Write down the next two terms, T_5 and T_6, in the sequence

$$1, 9, 25, 49, ...$$

Answer (*a*) T_5 = **81** [1]

T_6 = **121** [1]

(b) Write down an expression, in terms of *n*, for the n^{th} term in the sequence.

Answer (*b*) $T_n = (2n-1)^2$ [1]

(c) Write down the 50^{th} term in the sequence.

Answer (*c*) T_{50} = **9801** [1]

(d) Use your answer to part (c) to find

$$2 + 16 + 32 + ... + 784.$$

$$\begin{aligned} 2 + 16 + 32 + ... + 784 &= 2(1 + 8 + 16 + ... + 8 \times (50 - 1)) \\ &= 2 \times 9801 \\ &= 19602 \end{aligned}$$

Answer (*d*) **19602** [2]

For Examiner's Use

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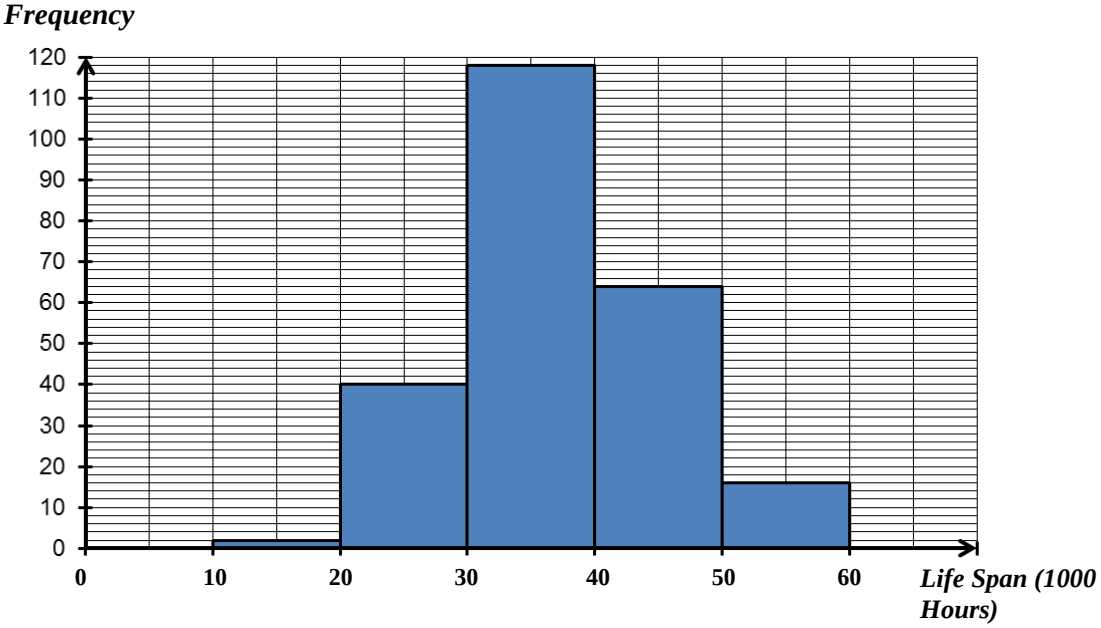
18 An electronic company *Qillips* is launching a new product, *Super-Saving LEDLight Bulb*. As part of their quality control measure, the company has randomly selected 240 light bulbs to estimate its life span.

The histogram below represents the results of the experiment.



(a) Complete the grouped frequency table of the life span of the light bulbs. [1]

Life Span (1000 hrs)	$0 \le x < 10$	$10 \le x < 20$	$20 \le x < 30$	$30 \le x < 40$	$40 \le x < 50$	$50 \le x < 60$
Frequency	0	2	40	118	64	16

(b) Write down the modal class.

Answer (b) $30000 \le x < 40000$ [1]

(c) Find the mean life span of the *Super-Saving LED Light Bulb*,

$$\text{mean} = \frac{5\times 0 + 15\times 2 + 25\times 40 + 35\times 118 + 45\times 64 + 55\times 16}{240} \times 1000$$

$$= 37200 hrs$$

Answer (c) 37200 hrs [2]

(d) If a *Super-Saving LED Light Bulb* with a life span shorter than 20 000 hours is considered as a faulty unit. Find the percentage of non-faulty light bulbs.

$$\text{percentage required} = \frac{240-2}{240} \times 100\%$$

$$= 99.2\%$$

Answer (d) 99.2% [2]

End of Paper