Mathematical Formulae

Compound interest

Total amount = \( P \left( 1 + \frac{r}{100} \right)^n \)

Mensuration

Curved surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC \) = \( \frac{1}{2} absinC \)

Arc length = \( r\theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\( a^2 = b^2 + c^2 - 2bc \cos A \)

Statistics

Mean = \( \frac{\sum fx}{\sum f} \)

Standard deviation = \( \sqrt{\frac{\sum f x^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2} \)
Answer all questions

1 (a) Calculate \( \frac{9.645^3}{\sqrt{6.55} + \sqrt{49.5}} \), giving your answer correct to 4 significant figures.

\[ \text{Answer (a)} \] \[ ........................ \] \[ 1 \]

(b) Simplify \( \left( \frac{1}{8x^{-6}y} \right)^\frac{2}{3} \times \left( \frac{y}{x^{-2}} \right)^0 \div \left( \frac{1}{y} \right)^\frac{1}{3} \).

\[ \text{Answer (b)} \] \[ ........................ \] \[ 2 \]

(c) The number 2003.09 can be written as

\[ 2 \times 10^3 + 3 \times 10^x + 9 \times 10^y. \]

Given that \( x \) and \( y \) are integers, find the values of \( x \) and \( y \).

\[ \text{Answer (c)} \] \[ x = ........ , y = ........ \] \[ 2 \]

2 (a) Calculate the difference of \( 4.6 \times 10^6 \) and \( 3.7 \times 10^5 \). Give your answer in standard form, to 2 significant figures.

\[ \text{Answer (a)} \] \[ ........................ \] \[ 2 \]

(b) The area of Singapore is about 710 \( \text{km}^2 \). Express the area in square metres, giving your answer in standard form.

\[ \text{Answer (b)} \] \[ ........................ \] \[ m^2 \] \[ 1 \]
<p>| | |</p>
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</table>
| 3 | **Given that** $p = 9.25 \times 10^{-99}$, $q = 4.76 \times 10^{-96}$, express $\frac{q}{p}$ in standard form.  

Answer ............................................ [1] |
| 4 | (a) Express $-m^2 + 6m - 13$ in the form $a(m - b)^2 + c$.  

Answer (a) ........................................... [2]  

(b) **Hence**, solve the equation $-m^2 + 6m - 13 = 0$, giving your answers correct to two decimal places.  

Answer (b) ............................................. [2] |
5  Expressed as the product of prime factors,
   \[ 132 = 2^2 \times 3 \times 11 \]  and  \[ 42 = 2 \times 3 \times 7 \]

   Use the results to find

   (a) the smallest integer, \( k \), such that \( 132k \) is a perfect cube.

   Answer (a) ............................... [1]

   (b) the smallest positive integer, \( n \), such that \( 42n \) is a multiple of 132.

   Answer (b) ............................... [1]

6  One solution of  \[ 2x^2 - 7 = a - 5x \] is  \( x = -4 \).

   Find

   (a) the value of \( a \),

   Answer (a) ............................... [1]

   (b) the other solution of the equation.

   Answer (b) ............................... [2]
7  (a) If \( A = \begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix} \) and \( B = \begin{pmatrix} 4 & 5 \\ 0 & -6 \end{pmatrix} \), evaluate \( C = A^2 + B \).

\[ \text{Answer (a) } \]

(b) A large cinema has a total of 20 theatres consisting of 2 theatres of type A, 3 of type B, 6 of type C and 9 of type D. The theatre has 3 classes of seat known as Economy, Executive and VIP. The table below shows the number of these seats in each of the 4 types of theatres.

<table>
<thead>
<tr>
<th></th>
<th>Economy</th>
<th>Executive</th>
<th>VIP</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>0</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>( B )</td>
<td>50</td>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>( C )</td>
<td>70</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>( D )</td>
<td>100</td>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

(i) Write down two matrices whose product shows the total number of seats in each class.

\[ \text{Answer (bi) } \]

(ii) Evaluate this product of matrices.

\[ \text{Answer (bii) } \]
8 (a) The diagram shows the graph of \( y = -(x - \frac{1}{2})^2 + k \).

(i) Write down the line of symmetry

\[ \text{Answer (ai)} \] …………………………………[1]

(ii) Given that the graph cuts the y-axis at \( L (0, 4) \), and cuts the x-axis at \( M (m, 0) \) and \( N (n, 0) \). Find the values of \( m \) and \( n \).

\[ \text{Answer (aii)} m = \] …………………….[1]  
\[ n = \] ……………………[1]

(b) The sketch represents the graph of \( y = x^n \). Write down a possible value of \( n \).

\[ \text{Answer (b)} \] ……………………………….. [1]

(c) Write down a possible equation for this graph.

\[ \text{Answer (c)} \] ……………………………….. [1]
<table>
<thead>
<tr>
<th>Question</th>
<th>Description</th>
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</table>
| 9        | Of the 38 students in a class, 23 have Line account and 15 have WeChat account. It is given that $\varepsilon = \{\text{students in the class}\}$, $A = \{\text{students who have Line account}\}$, and $B = \{\text{students who have WeChat account}\}$. Let $n(A \cap B) = x$. (a) Express $n(A \cup B)$ in terms of $x$.  

Answer (a) ……………………….. [2]  
(b) Find the smallest and largest possible values of $x$.  

Answer (b) Smallest possible value of $x$ is…………………………. [1]  
Largest possible value of $x$ is…………………………. [1] |
| 10       | Given that $a:b = 6:11$ and $a:c = 5:7$, find $a:b:c$.  

Answer ……………………………. [2] |
| 11 (a)   | Solve $-4 \leq -7x + 10 < 31$.  

Answer ……………………….. [2] |
(b) Write down the greatest and least integers which satisfy $-4 \leq -7x + 10 < 31$.

Answer (b) ............ and ............ [2]

12 An interior angle of a regular $n$-sided polygon is larger than its exterior angle by 50%. Find $n$.

Answer $n =$ ......................... [2]
13 The diagram shows two circles, $C_1$ and $C_2$, with centres $P_1$ and $P_2$ respectively. The two circles meet at $J$. $HK$ is a tangent to both circles. $AE$ is a diameter of the bigger circle.

(a) Given that the tangents of circle $C_1$ at $A$ and $C$ meet at $B$. $\angle ABC = 45^\circ$

Showing your working, calculate

(i) angle $ADC$

Answer $(a)(i)$ .................................. [2]

(ii) angle $AEC$

Answer $(a)(ii)$ .............................. [1]
(b) Circle $C_2$ passes through $J$, $F$ and $G$. $AFB$, $CGB$ and $HJK$ are tangents to the circle $C_2$.

(i) Given that $HB = 7$ cm, find the length of $BK$.

Answer (b)(i) ........................................... [2]

(ii) Hence, or otherwise, write down angle $BHK$.

Answer (b)(ii) ........................................... [1]
The vertices of triangle $ABC$ are $A(2, -1)$, $B(6, 5)$ and $C(0, 5)$.

(a) Find the area of triangle $ABC$.

Answer (a) ……………………….unit$^2$ [1]

(b) $CX$ is a line perpendicular to $AB$ and $X$ is a point on $AB$. Using your answer from part (a), find the length of $CX$.

Answer (b) ………………………. [3]

(c) Find $\angle ACB$.

Answer (c) $\angle ACB =$ ……………….. [3]
The diagram shows the distance-time graph for the first 50 seconds of a car’s journey. The car came to rest after 50 seconds.

(a) Find the speed of the car when \( t = 15 \)

Answer (a) \( \ldots \ldots \ldots \ldots m/s \) [1]

(b) Given that the car moved with a constant deceleration for the last 25 seconds. Find its deceleration for the last 25 seconds.

Answer (b) Deceleration for the last 25s is \( \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots m/s^2 \) [1]

(c) Given that the car also moved with a constant deceleration for the first 10 seconds, and the initial speed is 30 m/s.
On the axes given, sketch the speed-time graph for the car’s journey.

Answer (c) Speed (m/s) [3]
A scale map drawing shows the positions of the towns A, B and C. Given that \( AB = 11 \) cm, \( AC = 5 \) cm, \( B \) is due East of \( A \) and \( C \) is on a bearing of \( 055^\circ \) from \( A \).

**Answer (a), (bi),(bii) and (c)**

(a) Construct the triangular region \( ABC \). [2]

(b) In the \( \triangle ABC \), construct

(i) the perpendicular bisector of line \( AC \), [1]

(ii) the angle bisector of \( \angle CAB \), [1]

(c) Mark and label the point \( P \) inside \( \triangle ABC \) that is equidistant from the lines \( AC \) and \( AB \) and is also equidistant from town \( A \) and town \( C \). [1]
(d) Find the bearing of $A$ from $C$.

$Answer (d)$ Bearing of $A$ from $C$ = ………………… [1]

The map is drawn to a scale of 1:400 000.

(e) (i) Calculate the actual distance between town $A$ and town $B$.

$Answer (ei)$ ……………………………………………………………km [2]

(ii) Calculate the actual area of the triangular region $ABC$.

$Answer (eii)$ ………………………………………………………….$km$^2$ [2]
The terms $T_1, T_2, T_3, T_4$ of a sequence are given as follows:

$T_1 = 1 = 1$
$T_2 = 9 = 1 + 8$
$T_3 = 25 = 1 + 8 + 16$
$T_4 = 49 = 1 + 8 + 16 + 24$

(a) Write down the next two terms, $T_5$ and $T_6$, in the sequence

1, 9, 25, 49, ...

Answer (a) $T_5 =$ ........................................ [1]

Answer (b) $T_6 =$ ........................................ [1]

(b) Write down an expression, in terms of $n$, for the $n^{th}$ term in the sequence.

Answer (b) $T_n =$ ........................................ [1]

(c) Write down the $50^{th}$ term in the sequence.

Answer (c) $T_{50} =$ ........................................ [1]

(d) Use your answer to part (c) to find

$2 + 16 + 32 + ... + 784$.

Answer (d) ........................................ [2]
An electronic company Qillips is launching a new product, Super-Saving LED Light Bulb. As part of their quality control measure, the company has randomly selected 240 light bulbs to estimate its life span.

The histogram below represents the results of the experiment.

(a) Complete the grouped frequency table of the life span of the light bulbs. [1]

<table>
<thead>
<tr>
<th>Life Span (1000 hrs)</th>
<th>0 ≤ x &lt; 10</th>
<th>10 ≤ x &lt; 20</th>
<th>20 ≤ x &lt; 30</th>
<th>30 ≤ x &lt; 40</th>
<th>40 ≤ x &lt; 50</th>
<th>50 ≤ x &lt; 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>0</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Write down the modal class.

Answer (b) …………………………… [1]

(c) Find the mean life span of the Super-Saving LED Light Bulb,

Answer (c) …………………………… [2]

(d) If a Super-Saving LED Light Bulb with a life span shorter than 20 000 hours is considered as a faulty unit. Find the percentage of non-faulty light bulbs.

Answer (d) …………………………… [2]