

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} a b \sin C$$

$$\text{Arc length} = r \theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

1. (a) Express as a single fraction in its simplest form
$$\frac{3}{4a^2 - 25} - \frac{2}{5 - 2a}.$$
 [3]
- (b) Given that p is inversely proportional to the square of q .
Find the percentage decrease in p when the value of q is tripled. [3]
- (c) Given that $x + 2 = \frac{5y - 1}{x}$, express x in terms of y . [3]

2 Shane's monthly income was divided between savings, rental and other expenses in the ratio 5 : 4 : 6 respectively.
He saved \$1000 each month.

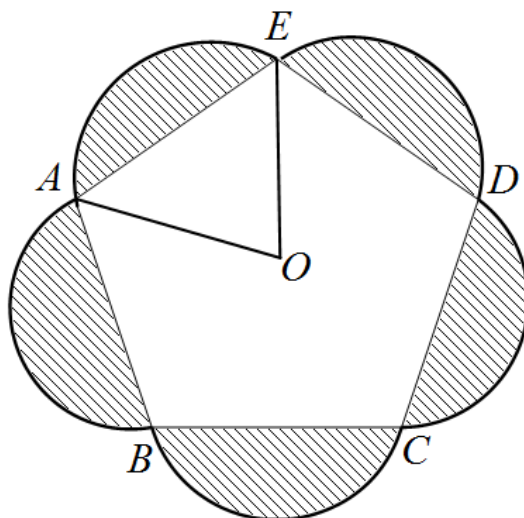
- (a) Find the amount he pays for his monthly rental. [1]
- (b) He deposits \$1000 into ABC bank that pays 1.45% per annum compound interest compounded monthly.
Calculate how much interest Shane will obtain after 3 years. [2]
- (c) The cash price of a sofa set is \$5888.
He buys the sofa set on hire purchase.
He pays a 30% deposit of the cash price followed by 24 monthly instalments of \$201.31.
Calculate the simple interest rate per annum. [3]
- (d) In 2013, he found a new job in London and his new income saw an increase of 23%. He will be paid £1900 per month.
Find the exchange rate in the form of 1 British Pound (£) to n Singapore Dollars (SGD). [2]
- (e) The total cost of water per month for a household is as follows:

	First 40 m ³ /month	Above 40 m ³ /month
Water Tariff	\$1.17 per cubic metre	\$1.40 per cubic metre
Water Conservation Tax	30% on amount payable for water used	45% on amount payable for water used

For the month of August, Shane used 45.2 m³ of water in a month. There is also a goods and services tax (GST) of 7% on the total water bill.
Calculate the amount Shane needs to pay for his water bill for the month of August. [3]

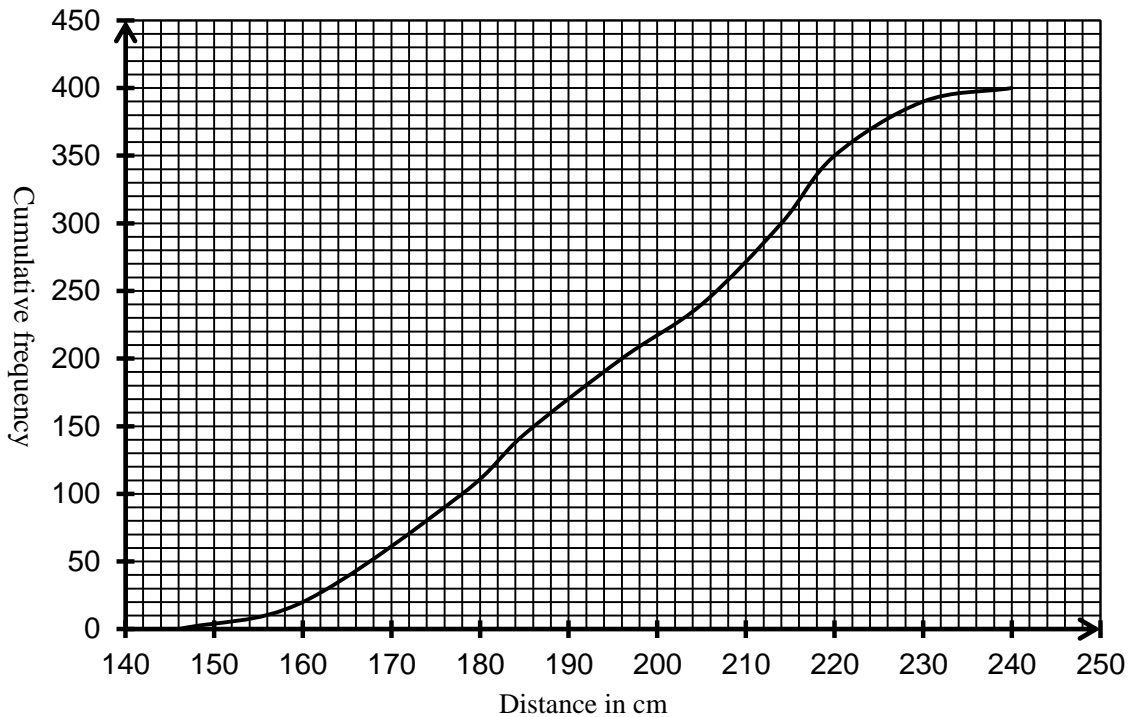
3. In 2012, Kai paid \$420 for x hours of tennis court rental.
In 2013, Kai is able to get 20 more hours of tennis court rental with \$450.
- (a) Write down an expression, in terms of x , for the amount paid by Kai for each hour of tennis court rental in 2012. [1]
- (b) Write down an expression, in terms of x , for the amount paid by Kai for each hour of tennis court rental in 2013. [1]
- (c) In 2013, it cost \$20 less to rent a tennis court for an hour.
Write down an equation in x to represent this information, and show that it reduces to
- $$2x^2 + 43x - 840 = 0. \quad [3]$$
- (d) Solve the equation $2x^2 + 43x - 840 = 0$. [3]
- (e) Find the maximum number of hours Kai can rent a tennis court such that his 2013 tennis court rental budget will not exceed \$450. [2]

4. The diagram below shows a badge which is made of five segments and a regular pentagon $ABCDE$ with centre O .
 $OA = 10$ cm.

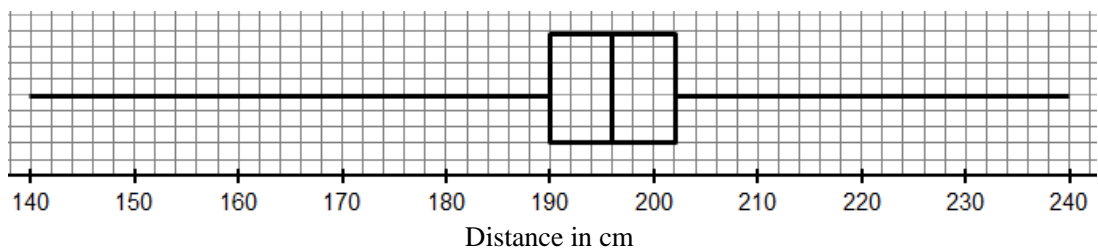


- (a) Show that angle AOE is 1.2566 radians. [1]
- (b) Calculate
- (i) area of the pentagon $ABCDE$, [3]
- (ii) area of the shaded region. [2]

5. The results of the standing broad jump segment of the NAPFA test by each of the 400 students in School *X* were measured. The cumulative frequency curve below shows the distribution of the distances jumped.

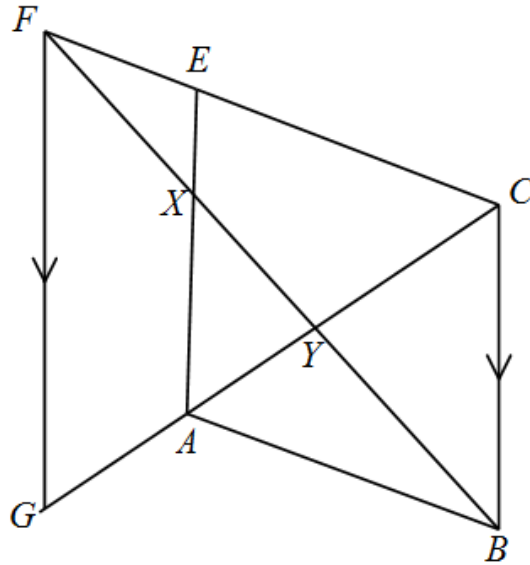


- (a) Estimate
- (i) the median distance jumped, [1]
 - (ii) the interquartile range, [2]
 - (iii) the minimum for a student to be in the top 10% of his cohort. [1]
- (b) Two students are chosen at random. Calculate the probability that both students' results are within the 50th to 70th percentile. [2]
- (c) The results of the standing broad jump segment of the NAPFA test by each of the 400 students in School *Y* were measured. The box-and-whisker plot below shows the distribution of their results.



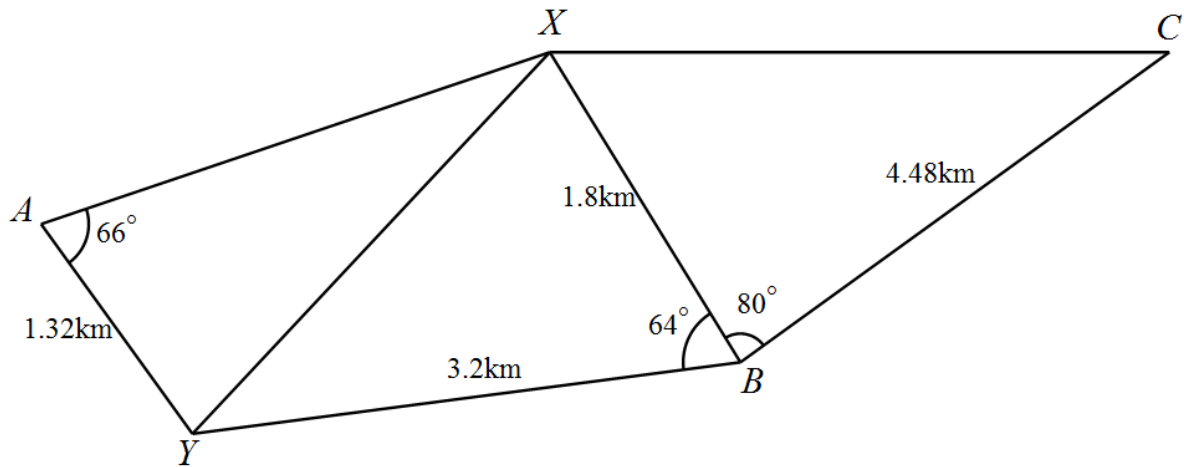
Describe how the cumulative frequency curve of School *Y* will differ from the given curve. Explain your answer. [2]

6. The diagram shows a rhombus $ABCE$ and an isosceles triangle CFG .
 The lines FG and BC are parallel.
 $FXYB$ is a straight line.



- (a) Prove that triangles ABC and CEA are congruent. [2]
- (b) Name the triangle that is similar to triangle FYG . Prove that they are similar. [2]
- (c) Given that $CY : YA : AG = 6 : 4 : 5$, find
- (i) $\frac{\text{Area of } \triangle AYB}{\text{Area of } \triangle CYF}$, [1]
- (ii) $\frac{\text{Area of } \triangle CBY}{\text{Area of } \triangle YAB}$, [1]
- (iii) $\frac{\text{Area of rhombus } ABCE}{\text{Area of } \triangle CFG}$. [2]

7. Terminals A , B and C use runway XY for flight departures and arrivals.
 $YB = 3.2$ km, $XB = 1.8$ km and $BC = 4.48$ km.
 Angle $YBX = 64^\circ$, angle $XAY = 66^\circ$ and angle $XBC = 80^\circ$



- (a) Show that the length of the runway XY is 2.9034 km. [2]
- (b) Grass is planted on the land plot $AYBX$ to strengthen the soil around the area. Calculate the area of the land plot $AYBX$. [3]
- (c) A plane approaches the runway XY at a descent angle (angle of depression) of 61° in order to make a safe landing.

Given that the plane is at an altitude of 3.2 km when it starts its descent at D , a point directly above X , calculate its horizontal distance from point X when it first touches down on the runway. [2]

- (d) A new road is built to connect airport terminal C to the road XB . Calculate the minimum length of the new road. [2]

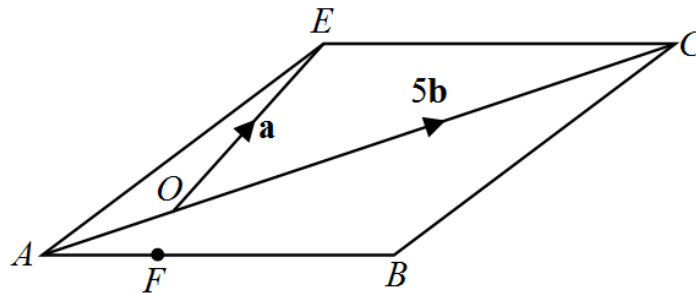
8 (a) X is the point $(3, 1)$.

It is also given that $\overrightarrow{XY} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$.

(i) Calculate $|\overrightarrow{XY}|$. [1]

(ii) Find the coordinates of Y . [2]

(b) **The diagram is not drawn to scale.**



The position vectors of C and E , relative to O , are $5\mathbf{b}$ and \mathbf{a} respectively.

$\overrightarrow{AC} = 6\overrightarrow{AO}$ and $AF : FB = 1 : 4$.

$ABCE$ is a parallelogram. AOC is the diagonal of the parallelogram.

(i) Express each of the following in terms of \mathbf{a} and \mathbf{b} ,

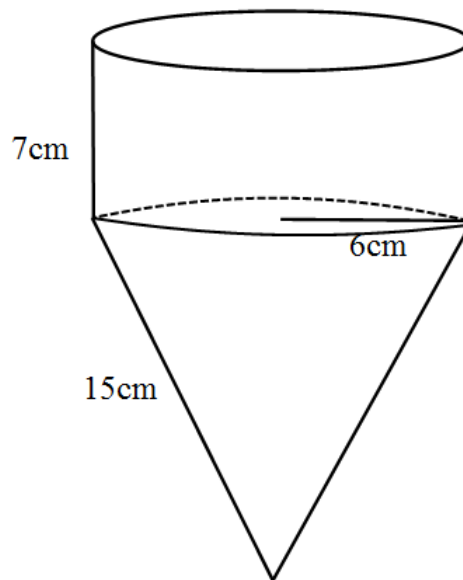
(a) \overrightarrow{AE} , [1]

(b) \overrightarrow{AF} , [2]

(c) position vector of F . [1]

(ii) Write down two facts about O , E and F . [2]

9.



The diagram above shows an open water container filled to its brim.
It is made up of a cone and a cylinder.
The cylinder has radius 6 cm and height 7 cm.
The cone has radius 6 cm and slant height of 15 cm.

- (a) Calculate
- (i) the height of the water container, [2]
 - (ii) outer surface area of the water container. [3]
- (b) Calculate the volume of the water container. [2]
- (c) It was observed that 0.013 litres of water flowed out when 3 spherical balls were submerged in the container.
Calculate the radius of the metallic spherical ball in centimetres. [3]

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10. (a) A survey was carried out to find the amount spent on lunch per day by each of the girls in a class.

The stem-and-leaf diagram below represents the results of the survey.

0		6	6	6			
1		2	5	5	5	8	9
2		1	1	1	1	1	
3		5					

Key: 0 | 5 means \$0.50

- (i) Calculate an estimate of
- (a) the mean amount of money spent on lunch, [1]
- (b) the standard deviation. [2]
- (ii) Find the median amount spent on lunch per day. [1]

The same survey was conducted on the same number of boys from the same class. The survey results are as shown below.

Mean = \$2.51
Median = \$1.80
Standard deviation = \$0.79

- (iii) By comparing these two groups of data, state with reason, which group of students generally spends more on lunch per day? [2]
- (b) Two dice were thrown together.
One of the die is a six-sided fair die numbered 1, 2, 3, 4, 5, 6. Another die is a four-sided fair die numbered 2, 3, 4, 5.
- (i) Draw the possibility diagram to show the outcome of the experiment. [1]
- (ii) Find, as a fraction in its simplest form, the probability that
- (a) the product of the outcome is 0, [1]
- (b) at least one of the number is a multiple of 3, [1]
- (c) the number on the four-sided die is greater than that on the six-sided die. [1]

11. **Answer the whole of this question on a sheet of graph paper.**

Your company is selling Lady KaKa's concert tickets.

The profits from the sale of the concert tickets are represented by the equation

$$P = -\frac{11}{500}S^2 + 8S - 84,$$

where \$ P is the profit from the sale of a concert ticket and \$ S is the selling price of a concert ticket.

Some corresponding values of P and S are given in the following table.

S	0	50	100	160	200	250	300	350	400
P	-84	a	496	632.8	636	541	336	21	-404

- (a) Calculate the value of a . [1]
- (b) Using a scale of 2 cm to represent \$50, draw a horizontal S -axis for $0 \leq S \leq 400$.
Using a scale of 2 cm to represent \$100, draw a vertical P -axis for $-404 \leq P \leq 650$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]
- (c) Use your graph to estimate the maximum profit and its corresponding selling price. [1]
- (d) By drawing a tangent, find the gradient of the curve at (300, 336). [2]
- (e) (i) Use your graph to find two solutions of $4000S - 42000 = 11S^2$. [2]
(ii) Explain what your answer to (e)(i) tells you about the selling price of Lady Kaka's concert ticket in order to make a profit. [1]

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