1. (a) (i) Factorise \(36q^3 - 16q\) completely. [2]

(ii) Simplify \(\frac{36q^3 - 16q}{6q^2 - 14q - 12}\). [2]

(b) Express as a single fraction in its simplest form \(\frac{2}{5x(x-2)} + \frac{x-1}{30-15x}\). [3]

(c) It is given that \(V = \frac{4\pi}{3}(a^3 - b^3)\).

(i) Evaluate \(V\) when \(a = 2.5\) and \(b = -1.8\). [1]

(ii) Express \(b\) in terms of \(V, a\) and \(\pi\). [2]

2. Eugene ran a marathon of 42 km.

For the first \(\frac{2}{3}\) of his run, his average speed was \(x\) km/h.

(a) Write down an expression, in terms of \(x\), for the time taken, in hours, for the first \(\frac{2}{3}\) of his run. [1]

He then reduced his speed by 2 km/h for the remaining part of the run.

(b) Given that the total time taken by Eugene to complete the run was \(4\frac{2}{5}\) hours, form an equation in \(x\) and show that it reduces to \(11x^2 - 127x + 140 = 0\). [3]

(c) Solve this equation and hence find his average speed for the last part of the run, giving your answer correct to 1 decimal place. [4]
3. Nomi and Adiel went for dinner at a Japanese restaurant in Singapore. The figure below shows a receipt for their meal in Singapore dollars. Part of the receipt was torn.

<table>
<thead>
<tr>
<th>J K Restaurant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Woodlands Drive</td>
</tr>
<tr>
<td>Singapore 123456</td>
</tr>
<tr>
<td>TEL: 67778888</td>
</tr>
<tr>
<td>GST NO MR-85000225-7 BIZ REG NO 53234707B</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date: 29/07/2016</th>
<th>Time: 18:00</th>
<th>Bill:A100029</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table: B3</td>
<td>Pax: 2</td>
<td></td>
</tr>
</tbody>
</table>

1. CHAWANMUSHI | 4.50 |
2. GREEN TEA | 3.00 |
8. SUSHI | 16.00 |
2. CHIC KATSU DON | 19.80 |

<table>
<thead>
<tr>
<th>SUBTOTAL</th>
<th>43.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% SVC CHG</td>
<td></td>
</tr>
<tr>
<td>7% GST</td>
<td></td>
</tr>
</tbody>
</table>

(a) Calculate the 10% service charge. [1]

(b) Calculate the 7% GST charge. Give your answer correct to 2 decimal places. [1]

(c) Adiel was still hungry and decided to order 2 more plates of sushi. Assuming they ordered the same price for each sushi, calculate the total amount they have to pay for their meals, inclusive of the service charge and GST. Give your answer correct to 2 decimal places. [3]

(d) As they did not have Singapore money to pay for their meals, they decided to pay in Japanese Yen. Given that the exchange rate was 100 Yen = 1.30 SGD, calculate the total amount they have to pay, giving your answer to the nearest Yen. [2]

(e) The price of one chawanmushi (before GST) was \( x\% \) less than the price in the year 2015. Write down an expression, in terms of \( x\), for the price of one chawanmushi in 2015. [2]
4 A fair blue die is numbered 2, 3, 4, 5, 15 and 17 while a fair red die is numbered 1, 9, 10, 11, 12 and 13.

(a) Draw a possibility diagram to show the possible outcomes when the two dice are tossed together. [2]

(b) Find, as a fraction in its simplest form, the probability that

(i) both dice have an even number, [1]

(ii) both dice have a prime number, [1]

(iii) the sum of the numbers on the dice is more than 17, [1]

(iv) at least one of the numbers on the dice is a multiple of 5. [1]

5 The diagram shows a circle \( BLCKJ \), centre \( O \). \( AE \) and \( AD \) are tangents to the circle. Angle \( JLB \) is equal to angle \( JBD \). Angle \( BAC = 50^\circ \), angle \( JLK = 22^\circ \), \( \angle KBD = 70^\circ \) and \( \angle BKL = 31^\circ \).

(a) Find, giving reasons for each answer,

(i) \( \angle JBK \), [1]

(ii) \( \angle KBL \), [2]

(iii) \( \angle KOL \). [1]
(iv) $\angle COB$ [2]

(b) An arc of a circle, with centre $A$, is drawn through the points $C$, $O$ and $B$.

The length of the arc $COB$ is $\frac{35}{18} \pi$ cm.

Calculate the area of the sector $ACOB$, leaving your answer in terms of $\pi$. [3]

6 A fruit seller bought $y$ apples for $21$.

(a) Find an expression, in terms of $y$, the cost, in cents, of each apple. [1]

(b) It was found that 3 of the apples were bad and were thrown away.

He sold each of the remaining apples for 4 cents more that he paid for it.

Write down an expression, in terms of $y$, the total sum he received from the sale of the apples. [1]

(c) He made a profit of $1.78$ from the sale of the apples.

Write down an equation to represent this information, and show that it simplifies to $4y^2 - 190y - 6300 = 0$. [3]

(d) Solve the equation $4y^2 - 190y - 6300 = 0$. [3]

(e) Hence, find the selling price of each apple. [1]
Two boats, $A$ and $B$ set off from a jetty $J$ at a bearing of $230^\circ$ and $117^\circ$ respectively. Upon reaching their respective destinations, the two boats are 284 m apart, boat $B$ is 170 m due south of a lighthouse $L$ and boat $A$ is at a bearing of $256^\circ$ from boat $B$.

(a) Show that $\angle JAB = 26^\circ$.

(b) Calculate

(i) the distance that boat $B$ travelled,

(ii) the distance of $L$ from boat $A$.

(c) The height of the lighthouse $L$ is 45 m. Calculate the greatest angle of depression of boat $B$ from the top of the lighthouse on its journey along the path $JB$.

8 In the diagram, $ABEF$ represents the rectangular sloping surface of a ramp. $ABCD$ is a square is on level ground. $CE$ and $DF$ are vertical lines. The gradient of the ramp is such that the vertical distance : horizontal distance is 1:4.

(a) Show that the angle of the ramp, angle $FAD$ is 14.0 correct to 1 decimal place.

(b) Given that $AF$ is 100 m, calculate

(i) $FD$,

(ii) $AC$,

(iii) $\angle EAC$.
(c) The ramp is to be made of concrete. The concrete consists of one part of cement and two parts of gravel. The cement costs 24 cents per cubic metre and the gravel costs 60 cents per cubic metre. Calculate the total cost to make the ramp. [4]

9 Answer the whole of this question on a sheet of graph paper.

The variables \( x \) and \( y \) are connected by the equation \( y = x + \frac{14}{x} - 6 \).

Some corresponding values of \( x \) and \( y \) are given in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>( a )</td>
<td>3</td>
<td>1.7</td>
<td>1.5</td>
<td>1.8</td>
<td>2.3</td>
<td>3</td>
<td>( b )</td>
</tr>
</tbody>
</table>

(a) Calculate the value of \( a \) and of \( b \). [1]

(b) Using a scale of 2 cm to represent 1 unit on both axes, draw the graph of \( y = x + \frac{14}{x} - 6 \) for \( 1 \leq x \leq 8 \). [3]

(c) Use your graph to find the value(s) of \( x \) where \( y = 3.5 \). [2]

(d) The line \( y = kx + c \) is a tangent to the curve \( y = x + \frac{14}{x} - 6 \) at point \( Q \). \( k = -1.5 \). By drawing a suitable straight line on the same axes, use your graph to find

(i) the coordinates of the point \( Q \), [1]

(ii) the value of \( c \). [1]

(e) By drawing a suitable straight line on the same axes, use your graph to

find the solutions of the equation \( \frac{4}{3}x + \frac{14}{x} - 12 = 0 \). [3]
10 The cumulative frequency graph below represents distribution of the times taken to travel from home to school on a particular day by a group of students in school A.

(a) Copy and complete the grouped frequency table of the time taken to travel from home to school on a particular day by the students.

<table>
<thead>
<tr>
<th>Time [x minutes]</th>
<th>0 ≤ x &lt; 20</th>
<th>20 ≤ x &lt; 40</th>
<th>40 ≤ x &lt; 60</th>
<th>60 ≤ x &lt; 80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Use the curve to estimate

(i) the median time, [1]

(ii) the interquartile range of the times. [2]
(c) The times taken by a group of 60 students from school B to travel from home to school on the same particular day had the same interquartile range as the times of the students in school A but a lower median. Describe how the cumulative frequency curve for the students in school B may differ from the curve for the students in school A. [1]

11 The first three terms in a sequence of numbers, \( T_1, T_2, T_3, \ldots \) are given below.

\[
\begin{align*}
T_1 &= 3^4 - 16 = 1 \quad 5 \quad (3^2 + 4) = 65 \\
T_2 &= 4^4 - 16 = 2 \quad 6 \quad (4^2 + 4) = 240 \\
T_3 &= 5^4 - 16 = 3 \quad 7 \quad (5^2 + 4) = 609 \\
\end{align*}
\]

(a) Find \( T_4 \). [1]

(b) Evaluate \( T_{50} \). [1]

(c) Find an expression, in terms of \( n \), for \( T_n \). [2]

(d) Hence, use your answer in (c) to find the three prime factors of 50609. [2]

12 The solid toy above consists of a cone with base radius \( r \) centimetres on top of a cylinder of radius \( r \) centimetres.

The height of the cylinder is twice the height of the cone.

The total height of the solid toy is \( H \) centimetres.
(a) Find an expression, in terms of $\pi$, $r$ and $H$, for the volume of the solid toy. Give your answer in its simplest form. [3]

(b) It is given that $r = 10\text{cm}$ and the height of the cone is 15 cm.

(i) Show that the slant height of the cone is 18.0 cm, correct to one decimal place. [1]

(ii) Find the circumference of the base of the cone. [1]

(iii) The curved surface area of the cone can be made into the shape of a sector of a circle with angle $\theta^\circ$. Show that $\theta$ is 200, correct to the nearest integer. [2]

(iv) Hence, or otherwise, find the total surface area of the solid toy. [3]
Answer

1(a)(i) \(4q(3q + 2)(3q - 2)\)

(ii) \(2q \left(\frac{3q - 2}{q - 3}\right)\)

(b) \(6 + x - x^2\) or \(\frac{(3 - x)(2 + x)}{15x(x - 2)}\)

(c)(i) \(V = 89.9\)

(ii) \(b = \sqrt[3]{\frac{3V}{4\pi}}\)

2(a) Time taken = \(\frac{28}{x}\) hours

(b) \(\frac{28}{x} + \frac{14}{x - 2} = \frac{22}{5}\)
\(\frac{28x - 56 + 14x}{x(x - 2)} = \frac{22}{5}\)
\(210x - 280 = 22x^2 - 44x\)
\(22x^2 - 254x + 280 = 0\)
\(11x^2 - 127x + 140 = 0\) (shown)

(c) \(x = \frac{127 \pm \sqrt{9969}}{22}\)
\(x = 10.311\) or \(x = 1.234\)

Average Speed = \(10.311 - 2 = 8.3\) km/h

3(a) \$4.33

(b) \$3.33

(c) \$55.67

(d) 4282 Yen

(e) \$ - \(\frac{4500}{100 - x}\)

4(a) | Red die | Blue die |
--- | --- | --- |
| 2 | x | x |
| 3 | x | x |
| 4 | x | x |
| 5 | x | x |
| 15 | x | x |
| 17 | x | x |

(b)(i) \(\frac{1}{9}\)

(ii) \(\frac{2}{9}\)

(iii) \(\frac{1}{3}\)

(iv) \(\frac{4}{9}\)
5(a)(i) \[ \angle JBK = \angle JLK \text{ (Angles in same segment)} \]
\[ = 22° \]

(ii) \[ \angle JBD = 70° - 22° = 48° \]
\[ \angle JLB = \angle JBD \text{ (Given)} \]
\[ \angle KBL = 180° - 31° - (22° + 48°) \text{ (\angle sum \triangle KBL)} \]
\[ = 79° \]

(iii) \[ \angle KOL = 2\angle KBL \]
\[ = 158° \]

(iv) \[ \angle COB = 360° - 90° - 90° - 50° \text{ (Angle sum of quad COBA)} \]
\[ = 130° \]

(b) \[ 50° \times \pi \times r = \frac{35\pi}{18} \]
\[ r = 7 \text{ cm} \]

Area of sector = \[ \frac{50°}{360°} \times \pi \times 7^2 \]
\[ = \frac{245}{36} \pi \text{ cm}^2 \]

6(a) \[ \frac{2100}{y} \]

(b) \[ (y - 3) \left( \frac{2100}{y} + 4 \right) \]

(c) \[ (d) y = 70 \text{ or } -22.5 \]
\[ (e) 34 \text{ cents or } \$0.34 \]

7(a) \[ 26° \]

(b)(i) \[ 135 \text{ m} \]

(b)(ii) \[ \text{LA= 365 m} \]

(c) \[ 16.5° \]

8(b)(i) \[ 24.3 \text{ m} \]

(ii) \[ 137 \text{ m} \]

(iii) \[ 10.0° \]

(c) \[ \$54784.10 \]

9(a) \[ a = 9, b = 3.75 \]

(c) \[ 1.85 \pm 0.1 \text{ and } 7.7 \pm 0.1 \]

(d)(i) \[ \text{Coordinates: (2.25, 2.5) \pm 0.1} \]
\[ c = 5.9 \pm 0.1 \]

(e) \[ x = 1.4 \text{ and } 7.6 \pm 0.1 \]

10(a)

<table>
<thead>
<tr>
<th>Time (in minutes)</th>
<th>0 \leq x &lt; 20</th>
<th>20 \leq x &lt; 40</th>
<th>40 \leq x &lt; 60</th>
<th>60 \leq x &lt; 80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>5</td>
<td>25</td>
<td>22</td>
<td>8</td>
</tr>
</tbody>
</table>

(b)(i) \[ 40 \text{ minutes} \]

(ii) \[ 19 \]

(c) \[ \text{The curve will shift more to the left.} \]

11(a) \[ T_4 = 6^4 \times 16 = 4 \times (6^2 + 4) = 1280 \]
(b) 7311600

(c) $T_n = (n+2)^4 = n(n+4)(n+2)^2 + 4$

OR

$T_n = (n+2)^4 = 4n^4 + 8n^3 + 24n^2 + 32n$

(d) 13, 17 and 229

12(a) $\frac{7\pi r^2 H}{9}$

(b)(i) 18.0

b(ii) 62.8

b(iii) 200

b(iv) 2760