

## O level strategies

The following tips on selected topics will help you through some questions in the sample papers for the O Level Elementary Mathematics Examination.

### Standard Questions

7. (a) If  $\mathbf{L} = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$  and  $\mathbf{M} = \begin{pmatrix} 6 & 0 \\ 3 & -4 \end{pmatrix}$ , evaluate  $\mathbf{N} = \mathbf{L} + \mathbf{M}$ .

Answer (a) ..... [2]

- (b) The table below shows the results for two football teams and the points awarded.

	Won	Drawn	Lost
Wochester	5	2	7
Stomapool	4	6	6

	Points
Won	3
Drawn	1
Lost	0

- (i) Find  $\begin{pmatrix} 5 & 2 & 7 \\ 4 & 6 & 6 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$ .

Answer (b)(i) ..... [2]

- (ii) Explain what your answer to (b)(i) represents.

Answer (b)(ii) ..... [1]

## Paper 1 Question 7. Topic: Matrices

Matrices questions usually rather standard. We just need to know how to operate on them. They can be tedious and requires lots of calculation but hardly difficult.

Students should know that  $\mathbf{M}^2$ , as required in Q7a, is not found by squaring each element in the Matrix M but by multiplying M by itself.

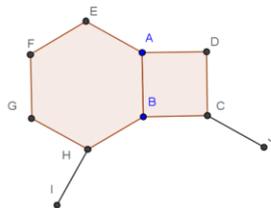
Two matrices can multiply each other, only if the number of columns in the 1<sup>st</sup> matrix is equal to the number of rows in the 2<sup>nd</sup> matrix.

When multiplying matrices, it is good to write down the orders (row by column) of the two matrices we are multiplying, as well as for the resulting matrix, if any. Then circle each row of the first matrix and each column of the second matrix to indicate that the elements of each row of the first matrix are to be multiplied by the elements

of each column in the second matrix. The products are then added to give the element of the corresponding row and column in the new matrix. For example, in Q7b(i), to get the element in the 2<sup>nd</sup> row and 3<sup>rd</sup> column of the resulting matrix, we find the sum of the products of the elements, 4, 6, 6, in 1st row of the first matrix with the elements, 3, 1, 0, in the 1st column of the second matrix.

Students are also expected to be able to interpret what each element in a matrix represents, as required in Q7b(ii). If we are clueless, we can gather hints and key words from the category titles (Wolchester, Stomapool) of the rows in the first matrix and the category titles of the columns (points) in the second matrix.

12.



*ABCD* is a square and *ABHGFE* is a regular hexagon.

*IHBCJ* is an incomplete regular polygon of  $n$  sides.

Find the value of  $n$ .

Answer ..... [2]

Paper 1 Question 12. Topic: Polygons.

Besides knowing the terminologies such as the names of the different polygons based on their number of sides, and that a regular polygon has equal measure for all angles and the same length for all sides, we need to remember 3 things:

- (i) Exterior angle + interior angle =  $180^\circ$  (not  $360^\circ$ )
- (ii) Sum of exterior angles =  $360^\circ$  (regardless of number of sides)
- (iii) Sum of interior angles =  $(n-2)180^\circ$ , where  $n$  is the number of sides.

Often, when we are asked to find the number of sides, as required in Q12, we should use (ii) Sum of exterior angles instead of (iii) sum of interior angles. To use (ii) Sum of exterior angles, we just need to find one exterior angle of the polygon, reflex angle *HBC*. We then divide the sum,  $360^\circ$ , by one exterior angle to get the number of exterior angle which is also the number of sides.

22. Given that  $A(5, 3)$  and  $B(-3, -2)$ .

- (a) Find the equation of the line  $AB$ .

Answer (a) ..... [2]

- (b) Find the value of  $k$  if the point  $\left(k, 1\frac{1}{2}\right)$  lies on the line  $AB$ .

Answer (b) ..... [1]

- (c) Find the length of  $AB$ .

Answer (c) ..... [1]

- (d) Find the equation of another line parallel to  $3x + y = 6$  and passing through point  $A$ .

Answer (d) ..... [2]

### Paper 1 Question 22. Topic: Coordinate Geometry.

Three points to remember for Coordinate Geometry, or straight line graphs:

- (i) Equation of Straight Line:  $y = mx + c$ ,  
where  $c$  is the  $y$ -intercept, i.e. where the graph cuts the  $y$ -axis.
- (ii) Gradient,  $m = \frac{y_1 - y_2}{x_1 - x_2}$
- (iii) Length of a line segment  $= \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$   
Remember it as the Pythagoras Theorem, hence the operation in the middle is +.

Using these three points, we will be able to find the equation of a straight line in all the three cases below.

- (i) Given gradient,  $m$  and  $y$ -intercept,  $c$ .

Simply substitute the values of  $m$  and  $c$  into Equation of Straight Line:  $y = mx + c$ .

- (ii) Given gradient,  $m$  and a point  $(x, y)$  which lies on the line.

To find the value of  $c$ , substitute the value of  $m$ ,  $x$  and  $y$  into Equation of Straight Line:

$$y = mx + c.$$

(ii) Given two points  $(x_1, y_1)$  and  $(x_2, y_2)$ , as in Q22a.

In this case, we have neither  $m$  nor  $c$ . Hence we have to first find  $m$  using the formula Gradient,  $m = \frac{y_1 - y_2}{x_1 - x_2}$ . Note that the coordinates of the same point should be in the same "column" when using this formula. Once we have  $m$ , we substitute the coordinates of any of the two points into Equation of Straight Line:  $y = mx + c$  to find the value of  $c$ .

Moderate,

3. The graph shows yearly passenger movements in Changi Airport, Singapore.



(image remixed from F1foreverF1's "Singapore Changi Airport - Passenger Movements (1998-2015).jpg", Wikimedia Commons, the free media repository)

Explain one way in which the graph is misleading.

Answer .....

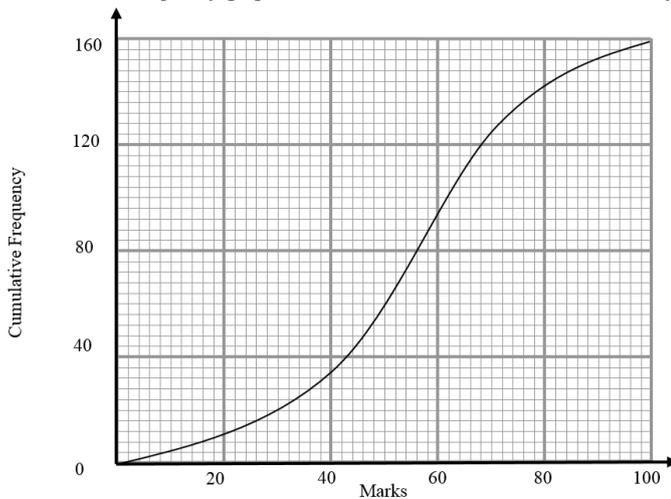
.....

..... [2]

Paper 1 Question 3 and 21. Topic: Statistics

In the new syllabus 4048 for Elementary Mathematics, students are expected to explain why a given statistical diagram leads to misinterpretation of data. One can get some clues on how to answer questions on misleading Statistical Diagrams by look at the axis-labels and scale of the axes. For diagrams without axes like pictogram and pie chart, perhaps even for dot diagrams, you may like to look at the size of the icons, dots or sectors in relation to the value they represent.

21. The cumulative frequency graph below shows Mathematics marks scored by 160 students.



- (a) Using the above cumulative frequency curve, find the
- (i) the percentage of students who scored less than 50 marks.

*Answer (a)(i)* ..... [1]

- (ii) number of students who scored distinctions if the distinction mark is 74 and above.

*Answer (a)(ii)* ..... [1]

- (iii) inter-quartile range.

*Answer (a)(iii)* ..... [1]

- (b) In a Science examination, the same students had a median mark of 51. The inter-quartile range for the Science examination is 22 marks. Which paper, Mathematics or Science, was an easier paper? Give a reason for your answer.

*Answer (b)* ..... [1]

Also, the construction of statistical charts is no longer required. Yes! So do not spend time perfecting skills on how to draw the charts. Instead, students should focus on interpretation of data and using them to compare two sets of data, as required in Q21b.

When presented with two sets of data, we compare their

1. central tendency, which is like the representative of the group of data, using mean or median, and
2. spread, using standard deviation (with mean) and interquartile range (with median).

A higher standard deviation or interquartile range indicates that the data has a wider spread while the central tendency represents the data, telling us whether the group is heavier or lighter, taller or shorter, scores higher or lower, etc.

1. (a) Express  $\frac{2}{9a^2 - 16} - \frac{3}{4 - 3a}$  as a single fraction in its simplest form. [3]
- (b) Given that  $y$  is inversely proportional to the square of  $x$ .  
Find the percentage decrease in  $y$  when the value of  $x$  is doubled. [3]
- (c) Given that  $x = \frac{3y + 2}{x - 1}$ , express  $x$  in terms of  $y$ . [3]

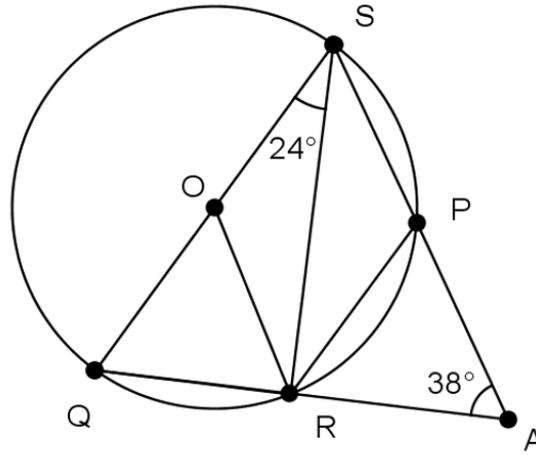
Paper 2 Question 1. Topic: Algebra.

When simplifying or operating (+, −, ×, ÷) on algebraic fractions, as required in Q1a, the key is to factorise. For Addition and Subtraction, we factorise the denominators completely before finding the common denominator. For simplification, multiplication and division, we factorise everything to change the operation to multiply so that we can “cancel” the common factors.

To factorise, remember these:

1. “take out” common factor(s).  
once we “take out” all common factors, look at the number of terms to be factorise. If we have
2. two terms in the form of  $a^2 - b^2$ , factorise into  $(a + b)(a - b)$ , as in the case of the denominator of Q1a;
3. three terms in quadratic form, e.g.  $3x^2 - 5x + 2$ , we use the cross method or some uses the box-method.
4. four terms, we factorise by grouping.

9. In the diagram,  $QS$  is a diameter of the circle, centre  $O$ .  $AQR$  and  $APS$  are straight lines. Given that  $\angle QSR = 24^\circ$  and  $\angle QAP = 38^\circ$ ,



Stating clearly your reasons, find

- |     |                |     |
|-----|----------------|-----|
| (a) | $\angle SQR$ , | [1] |
| (b) | $\angle ROS$ , | [1] |
| (c) | $\angle SPR$ , | [2] |
| (d) | $\angle PRQ$ , | [3] |
| (e) | $\angle PRO$ . | [1] |

Paper 2 Question 9. Topic: Angle Properties of Circles.

When tackling any questions with diagram, one should read the information and make sure that all information are reflected on the diagram before beginning to answer the questions.

Besides know all the symmetrical properties and angle properties of circles, students should do some preparation work on the diagram before attempting to find the values of required angles.

The preparation involves marking out two things as follows.

1. All right angles, arising from
  - (i) Tangent perpendicular to radius at point of contact, and
  - (ii) Angle in semicircle. Look out for diameter to identify this angle.
2. All equal lengths.
  - (i) All radii (plural of radius) are equal.
  - (ii) The pairs of tangents from an external point are equal.

Do note that equal lengths will make isosceles triangles so look out for them.

Challenging)

8. **Answer the whole of this question on a sheet of graph paper.**

The variables  $x$  and  $y$  are connected by the equation  $y = x - 2 + \frac{8}{x}$ . The table below shows some corresponding values of  $x$  and  $y$ . The values of  $y$  are given correct to one decimal place where appropriate.

$x$	1	1.5	2	3	4	5	6	7	8	9
$y$	7.0	4.8	4.0	3.7	4.0	4.6	5.3	$k$	7.0	7.9

- (a) Calculate the value of  $k$ , giving your answer correct to 1 decimal place. [1]
- (b) Taking 2 cm to represent 1 unit on both  $x$ -axis and  $y$ -axis, draw the graph of  $y = x - 2 + \frac{8}{x}$  for  $1 \leq x \leq 9$ . [3]
- (c) Use your graph to estimate
- (i) the value of  $x$  for which  $y$  is a minimum, [1]
- (ii) the of values of  $x$  for which  $y = 5$ . [1]
- (d) By drawing a tangent, find the gradient of the curve at  $x = 3.5$ . [2]
- (e) By drawing a suitable straight line on the same axes, solve the equation

$$4\left(x - 2 + \frac{8}{x}\right) = x + 20. \quad [3]$$

Paper 2 Question 8. Topic: Graphs

For the graph question which is to be done on a single piece of graph paper, students should write either all their answers at the blank side of the graph paper or all their answers on the graph side of the paper, gathered at one spot rather than all over the graph. Some schools make their students leave the first two centimetres of their graph paper blank for writing the answers on.

The graph question is really not very difficult and rather routine and can be perfected through practice. The only challenge, perhaps, is the last part(s) where students are required to draw a suitable straight line, as required in Q8e, to solve a given equation. To find the equation of that suitable straight line, we make the express for  $y$  in equation of the graph, in this question,  $x - 2 + \frac{8}{x}$ , the subject. Then the expression for  $y$  of the suitable

straight line is simply what's on the other side of the equation, in this case,  $y = \frac{x + 20}{4}$ .

Note that you should spend about 15 to 18 minutes on the graph question.

11. Here is some information about a portable vertical storage tank for bulk powder products.

Capacity	62.3 m <sup>3</sup> #
Capacity (Cement)	96 000 kg*

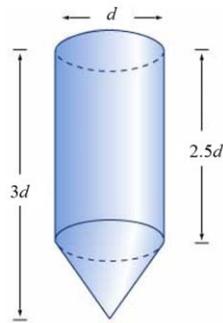
#Safety Information: The capacity of the tank is 90% of the total volume of the interior of the tank.

\*Cement can weigh between 1 410 – 1 506 kg/m<sup>3</sup> depending on how aerated it is.



(Image taken from: <http://www.cementsilos.com>)

- (a) Mass of one such empty storage tank should not be more than one-third the mass of its contents. What is the maximum possible mass of an empty storage tank? [1]
- (b) In this question the interior of the storage tank can be modelled as a cylinder with an inverted cone at the bottom with diameter  $d$  mm. Work out the value of  $d$ . [4]



- (c) Ali wants to fill the tank with 96 000 kg of cement. Comment on whether this is possible and whether it is advisable. Justify your comment with calculations. [3]

## Paper 2 Question 11. Topic: Applying mathematics to a real-world scenario

The last question of Paper 2, Elementary Mathematics, syllabus 4048, will focus specifically on applying mathematics to a real-world scenario. This question will most likely come from the newly added syllabus content on solving problems based on real-world context (including floor plans, surveying, navigation, etc.) using geometry. Students are expected to interpret the solution in the context of the problem.

To tackle this type of problems, it'll be good if we can imagine the context of the problem. We then write the dimensions given onto the diagram of the model. The key is to link all information given by looking at their units. For example, to answer Q11c, we need to know that we are comparing mass (kg), and we have the volume (m<sup>3</sup>), so we should be looking at the density (kg/m<sup>3</sup>).

## Preparation for exam day

### 1. Practice makes Perfect

To do well for Mathematics, we not only need to practise, but we need to get feedback to ensure that what we practise is correct. Hence, it is important to check your answers against the solutions and correct your mistakes.

**2. Rest early the night before**

Have a good night rest the night before the Mathematics papers so that you have a clear mind to tackle those non-routine and real-world context questions which require more thinking and analysis.

**3. Bring your Mathematical Instruments**

Pack your pencil case the day before the examination, making sure that you have the following:

1. Calculator(s) of approved models
2. Black or blue ball point pens
3. Working mechanical pencils with lead
4. Ruler
5. Eraser
6. Compass with sharp pencil
7. Protractor
8. Set Squares
9. Curve ruler

**4. Time Management**

Bring a watch or a small clock and keep track of the time as you do your paper. For the Mathematics papers, you should not spend more than 1.5 minute for each mark. Skip the difficult questions and come back to them later.

**5. Paper Management for Paper 2**

You should begin answering each question on a new page for Paper 2 so that you do not have to worry about how much space to leave if you need to skip certain parts of the question and yet have ample space when you come back later.

With that, all the best to all candidates sitting for the new syllabus O Level Elementary Mathematics Examinations.